Statistical Process Control

Kvalitetsflow 2015

Suggested Program

Day 1		
08.00-09.50	8.00-8.30 8.30-8.55 9.00 – 9.50	Welcome – Presentation and introduction Over view of SPC – The basics The reason for SPC – flow chart Variation
Coffee		
10.10.-12.10	10.10-11.00 11.05-12.10	SPC – Cpk – Six Sigma – short discussion What type of chart in different situations
Lunch		
12.40-15.00	12.45-13.15 13.20-14.15 14.20-14.40 14.50-15.00	Setting up SPC Systems Checklist : Ready to measure ? Discussion – What do need the most ? Planning for day 2

Suggested Program

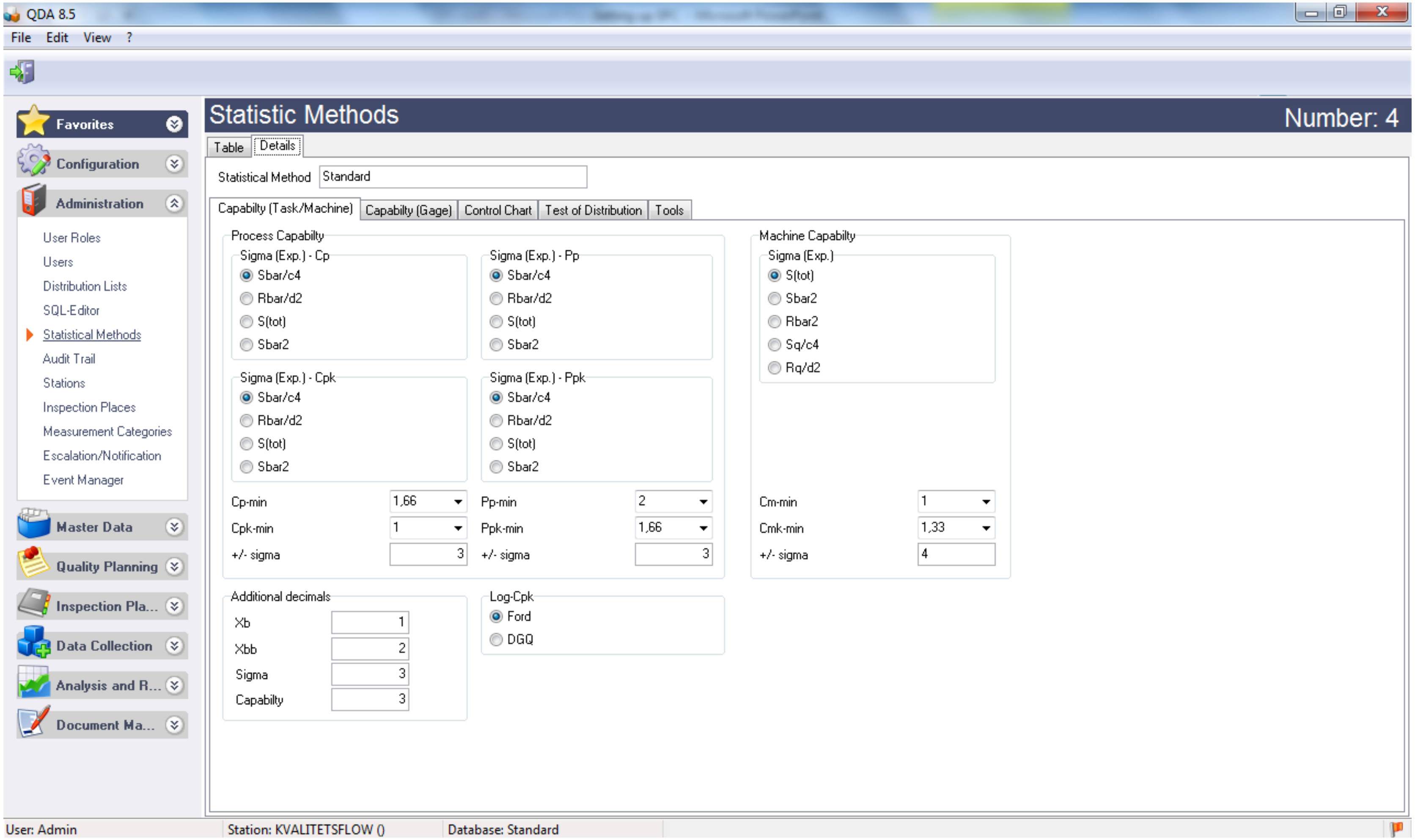
Day 2		
08.00-09.50	8.00-8.30 8.30-8.55 9.00 – 9.50	Setting up SPC in QDA Make QDA fit to your data Case: try it on your data
Coffee		
10.10.-12.10	10.10-11.00 11.05-12.10	Case: try it on your data Analysis and reporting
Lunch		
12.40-15.00	12.45-13.15 13.20-14.15 14.20-14.40 14.50-15.00	Analysis and reporting Additional theory? Analysis on your data – setting up reports Evaluation and actions agreed

Training in SPC and Setting up SPC in QDA

Setting up a SPC in general

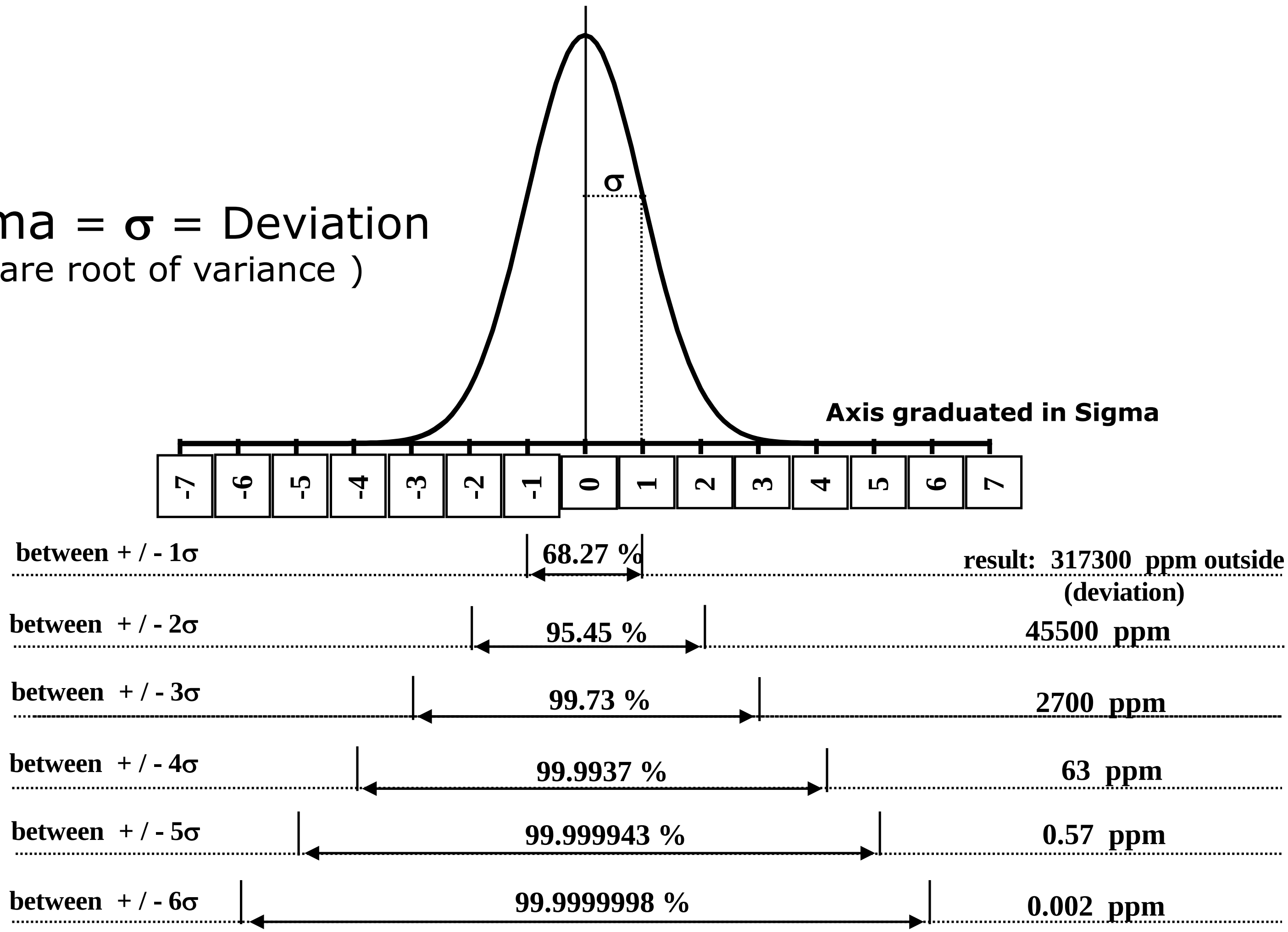
- Proces variables – how to find the relation between the proces variables and our SPC measure system.
- First things first – MSA (Measure system analysis) to approve the system before we collect all data make decision on ”shaky ground”
- Collecting basic information about the proces
  - USL, LSL
  - UCL, LCL
  - CP and Cpk values and targets

Setting up SPC i QDA

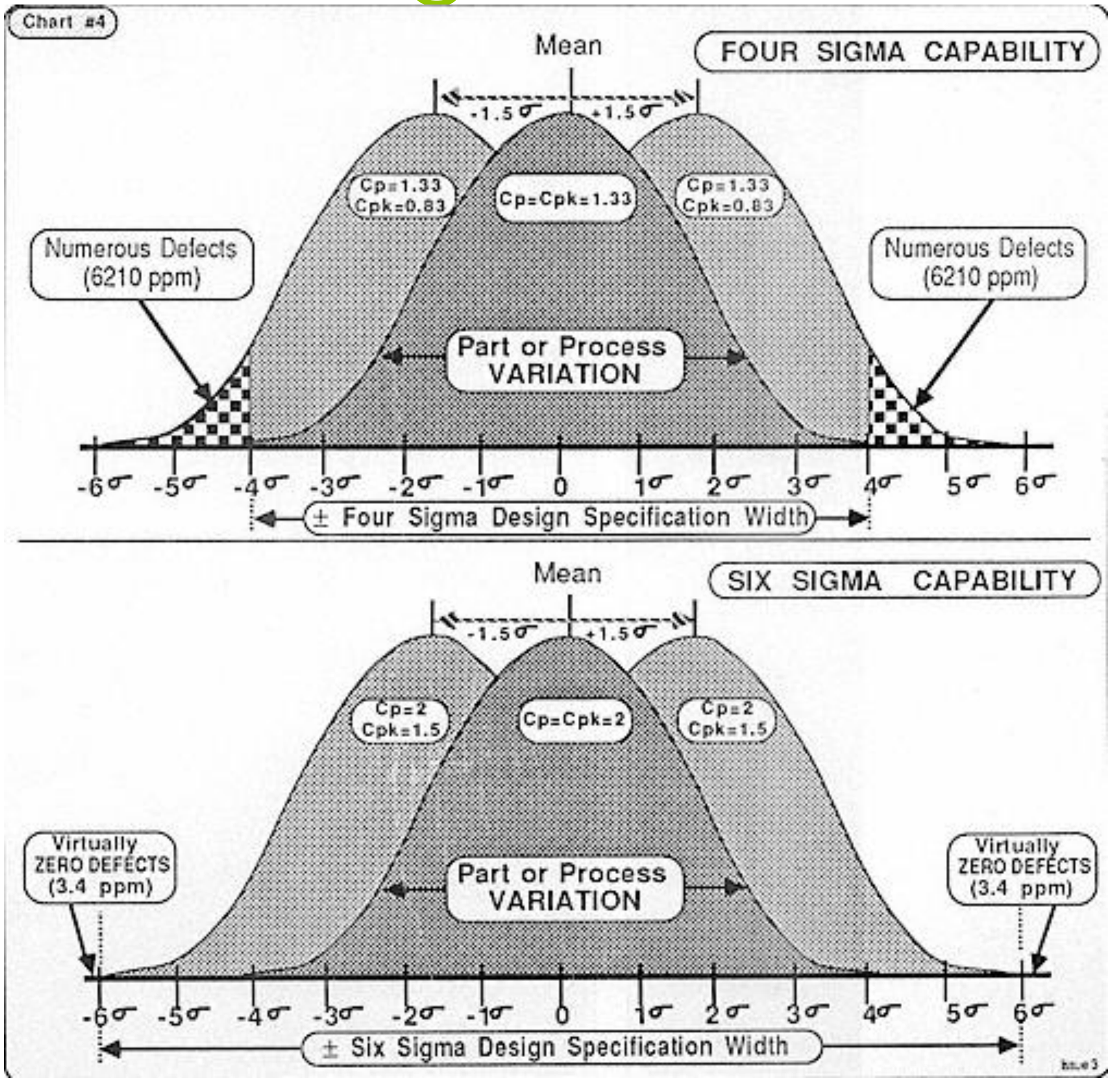


Six Sigma as a Metric

Sigma = σ = Deviation
(Square root of variance)



Effect of 1.5 Sigma Process Shift



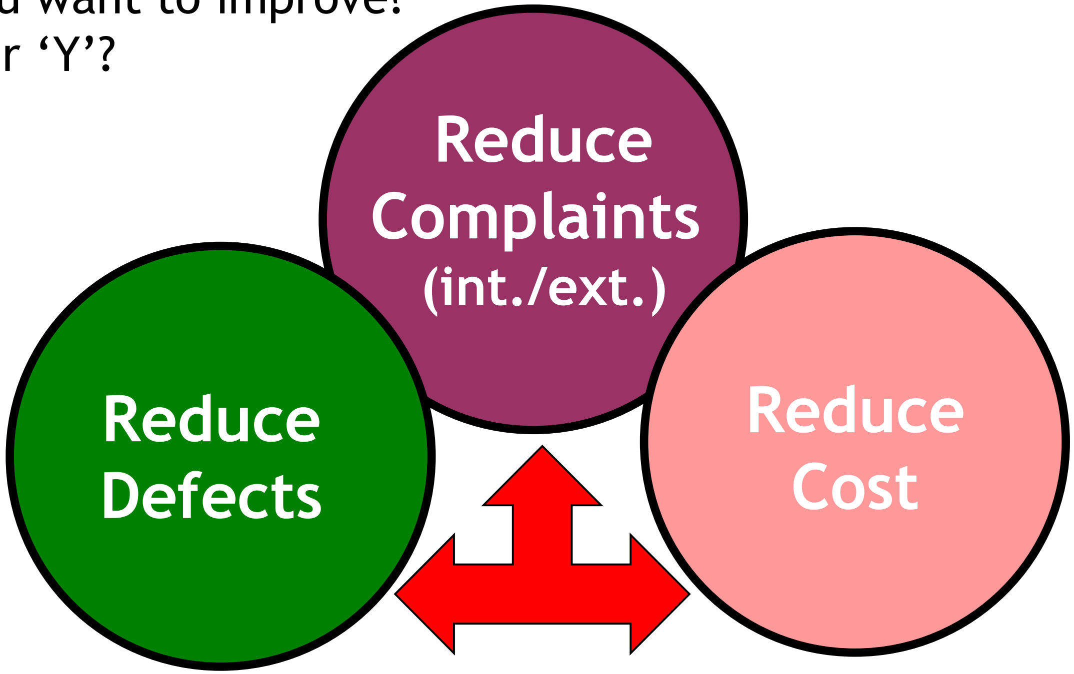
Drift and combinations of PPM and sigma

- part per million.
- Green, indicates data that are normally used

Sigma Drift	3σ	3.5σ	4σ	4.5σ	5σ	5.5σ	6σ
0σ	2,700	465	63	3.4	0.57	0.034	0.002
0.25σ	3,577	666	99	12.8	1.02	0.1056	0.0063
0.5σ	6440	1382	236	32	3.4	0.71	0.019
0.75σ	12228	3011	665	88.5	11	1.02	0.1
1σ	22832	6443	1350	233	32	3.4	0.39
1.25σ	40111	12201	3000	577	88.5	10.7	1
1.5σ	66803	22800	6200	1350	233	32	3.4
1.75σ	105601	40100	12200	3000	577	88.4	11
2σ	158,700	66800	22800	6200	1300	233	32

Problem Definition

- What do you want to improve?
- What is your ‘Y’?



What are the ***Goals?***

Problem Definitions need to be based on quantitative facts supported by analytical data.

Map the Process

Identify the variables - ‘x’

Measure the Process

Understand the Problem -
’Y’ = function of variables -’x’
Y=f(x)

To understand where you want to be, you need to know how to get there.

Control Phase

How to create a Control Plan: (one suggestion...)

- Select Causal Variable(s). Proven vital few X(s)
- Define Control Plan
  - 5Ws for optimal ranges of X(s)
- Validate Control Plan
  - Observe Y
- Implement/Document Control Plan
- Audit Control Plan
- Monitor Performance Metrics

Control Phase

Control Plan Tools:

- Statistical Process Control (SPC)

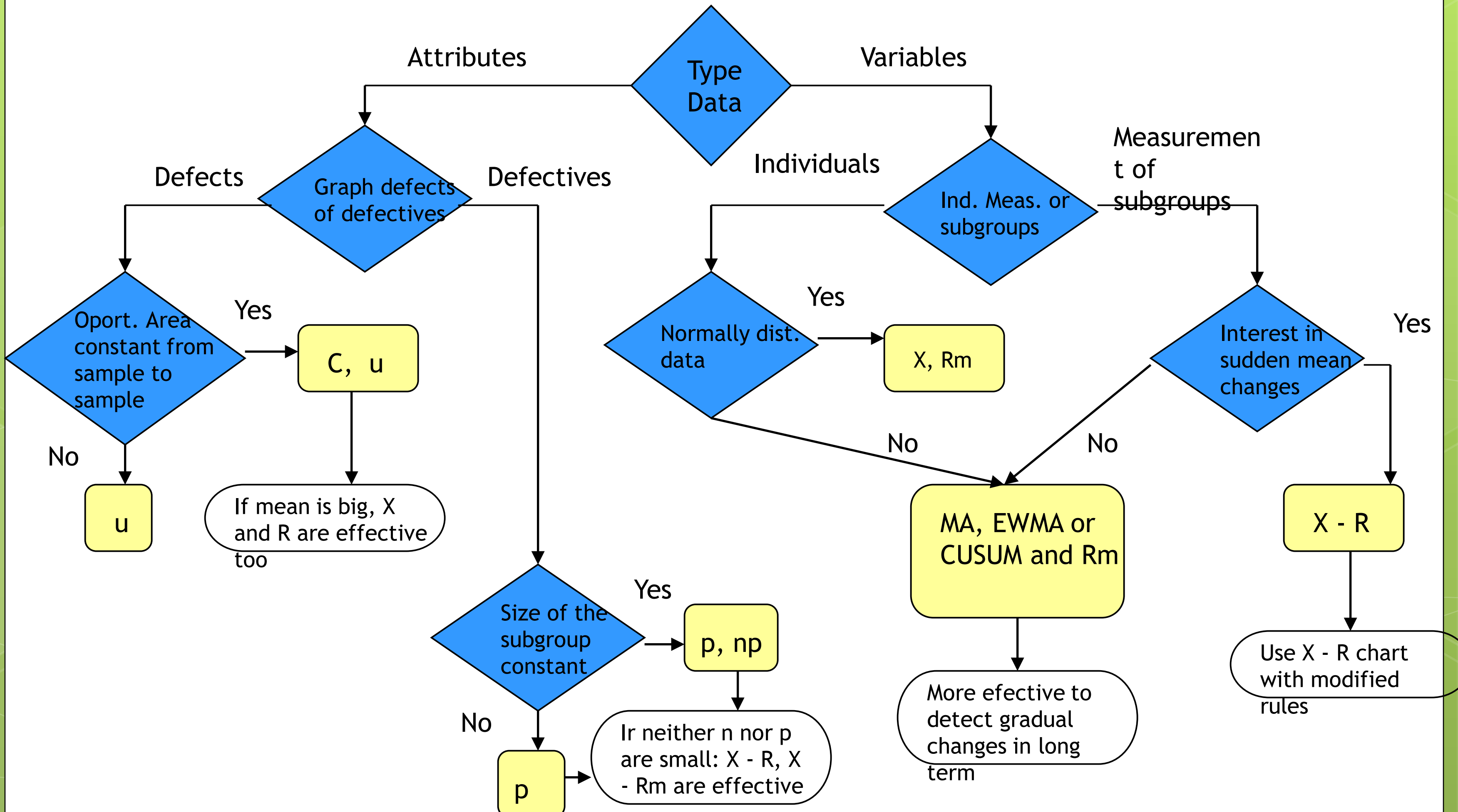
  Used with various types of distributions

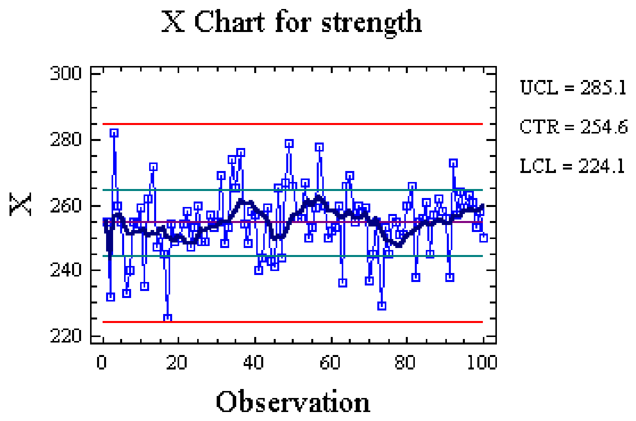
  Control Charts

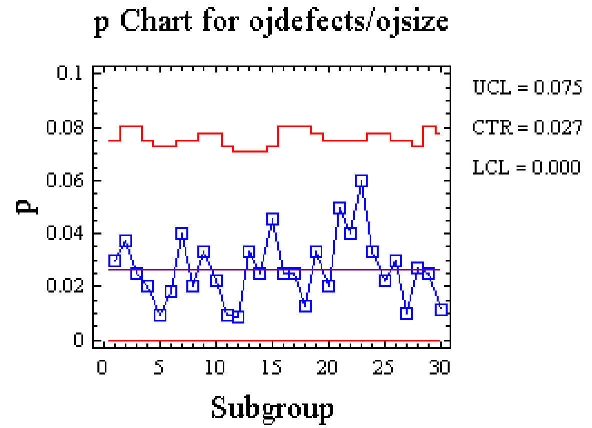
  - Attribute based (np, p, c, u). Variable based (X-R, X)
  - Additional Variable based tools
    - -PRE-Control
    - -Common Cause Chart (Exponentially Balanced Moving Average (EWMA))

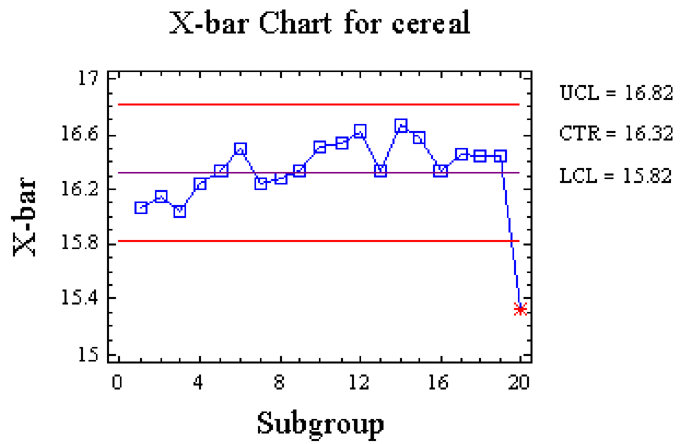
Control Phase

How do we select the correct Control Chart:




X Chart for strength
300
280
260
240
220
X
0
20
40
60
80
100
Observation
UCL = 285.1
CTR = 254.6
LCL = 224.1


p Chart for ojdefects/ojsize
0.1
0.08
0.06
0.04
0.02
0
p
0
5
10
15
20
25
30
Subgroup
UCL = 0.075
CTR = 0.027
LCL = 0.000


X-bar Chart for cereal
17
16.6
16.2
15.8
15.4
15
X-bar
0
4
8
12
16
20
Subgroup
UCL = 16.82
CTR = 16.32
LCL = 15.82

Statistical Process Control (SPC)

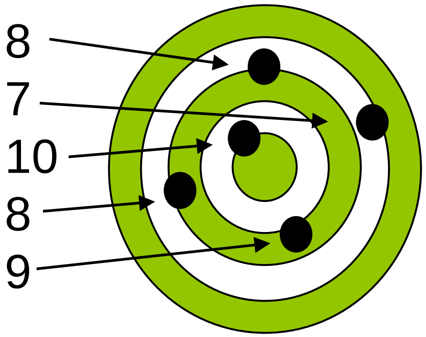
- Invented by Walter Shewhart at Western Electric
- Distinguishes between
  - common cause variability (random)
  - special cause variability (assignable)
- Based on repeated samples from a process

Statistical Process Control (SPC)

- A methodology for monitoring a process to identify special causes of variation and signal the need to take corrective action when appropriate
- SPC relies on control charts

Variability

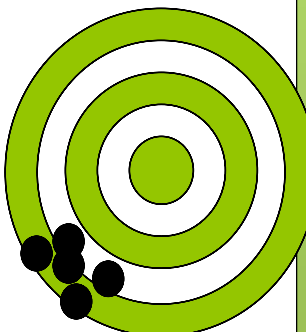
- Deviation = distance between observations and the mean (or average)



Emmett

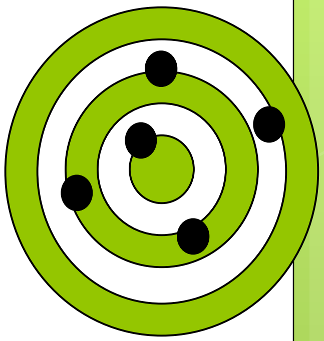
- Results for "Emmett"

	Observations	Deviations
	10	10 - 8.4 = 1.6
	9	9 - 8.4 = 0.6
	8	8 - 8.4 = -0.4
	8	8 - 8.4 = -0.4
	7	7 - 8.4 = -1.4
averages	**8.4**	**0.0**


Jake

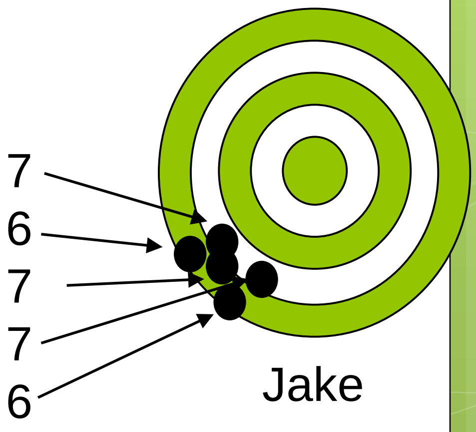
Variability

- Deviation = distance between observations and the mean (or average)



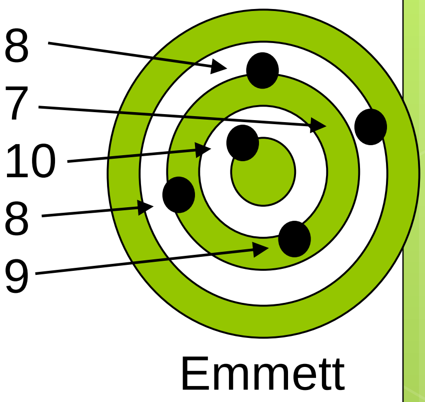
- Results for "Jake"

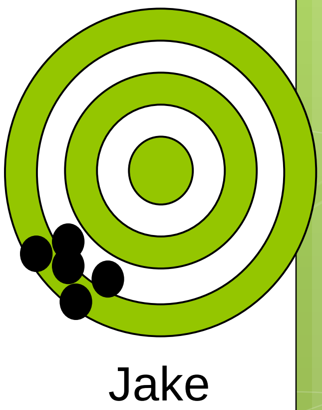
	Observations	Deviations
	7	7 - 6.6 = 0.4
	7	7 - 6.6 = 0.4
	7	7 - 6.6 = 0.4
	6	6 - 6.6 = -0.6
	6	6 - 6.6 = -0.6
averages	**6.6**	**0.0**



Variability

- Variance = average distance between observations and the mean squared



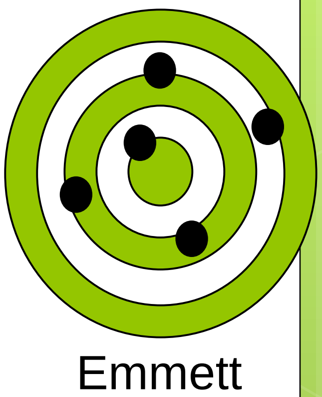


	Observations	Deviations	Squared Deviations
	10	10 - 8.4 = 1.6	2.56
	9	9 - 8.4 = 0.6	0.36
	8	8 - 8.4 = -0.4	0.16
	8	8 - 8.4 = -0.4	0.16
	7	7 - 8.4 = -1.4	1.96
averages	**8.4**	**0.0**	**1.0**

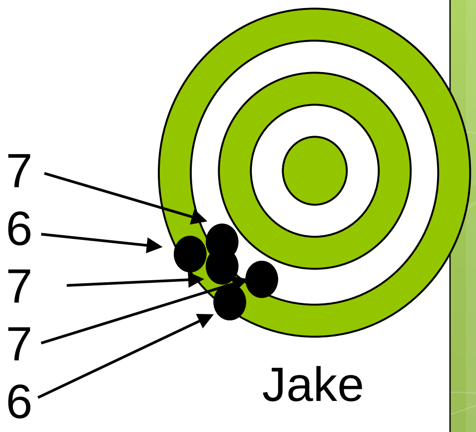
Variance

Variability

- Variance = average distance between observations and the mean squared

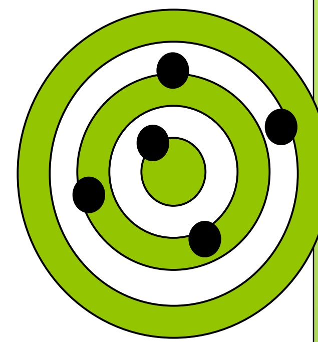


	Observations	Deviations	Squared Deviations
	7		
	7		
	7		
	6		
	6		
averages			

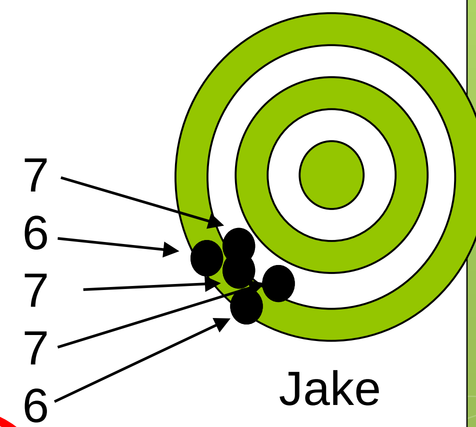


Variability

- Variance = average distance between observations and the mean squared



	Observations	**Deviations**	Squared Deviations
	7	**7 - 6.6 = 0.4**	0.16
	7	**7 - 6.6 = 0.4**	0.16
	7	**7 - 6.6 = 0.4**	0.16
	6	**6 - 6.6 = -0.6**	0.36
	6	**6 - 6.6 = -0.6**	0.36
averages	**6.6**	**0.0**	**0.24**

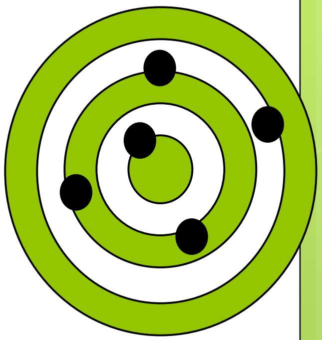


Variance

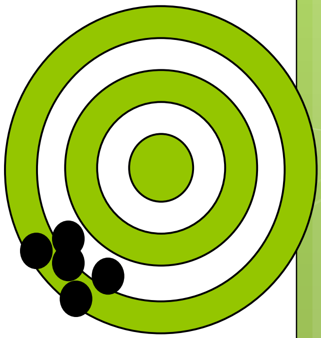
Variability

- Standard deviation = square root of variance

	Variance	Standard Deviation
Emmett	1.0	1.0
Jake	0.24	0.4898979

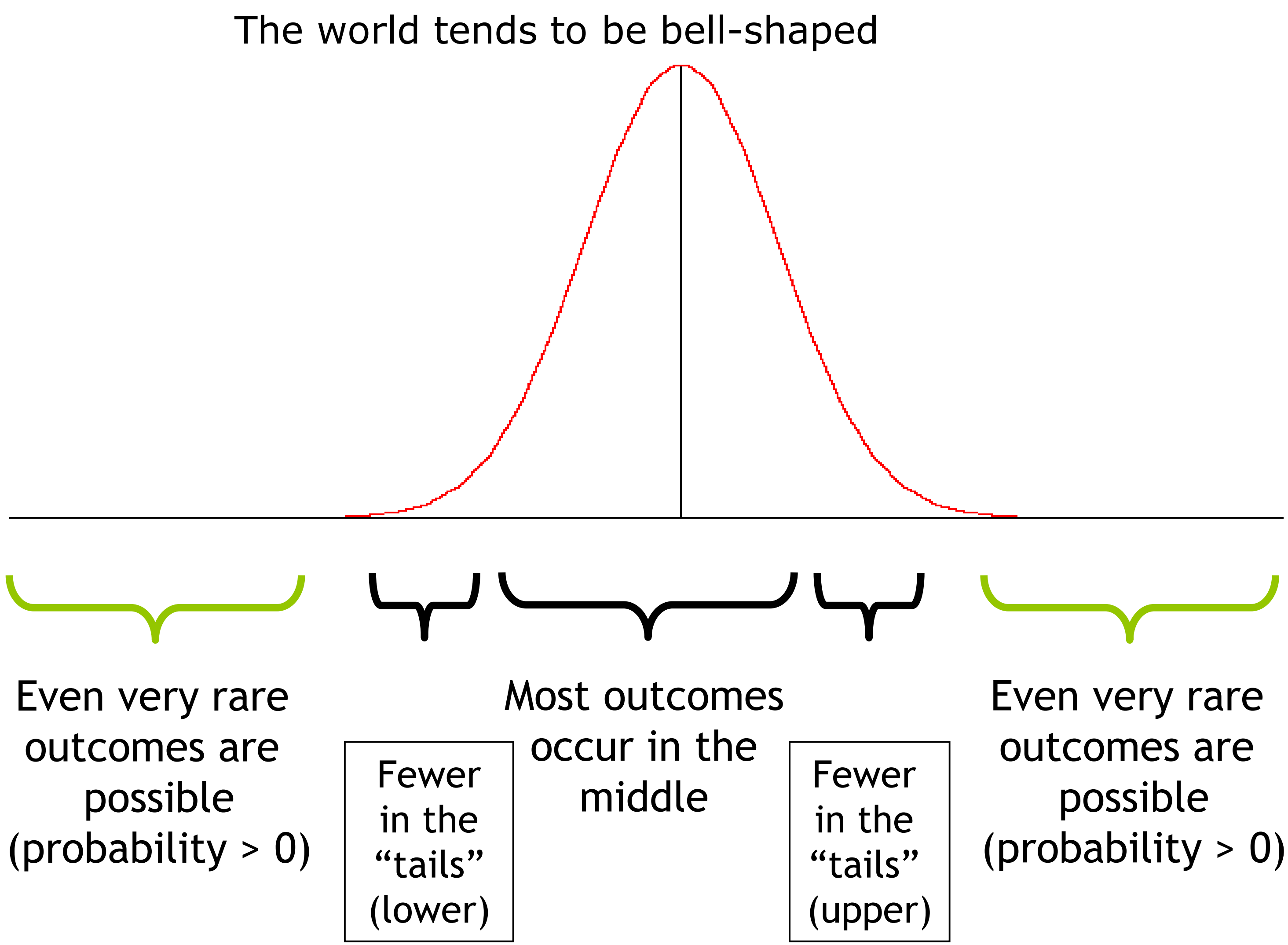


Emmett



Jake

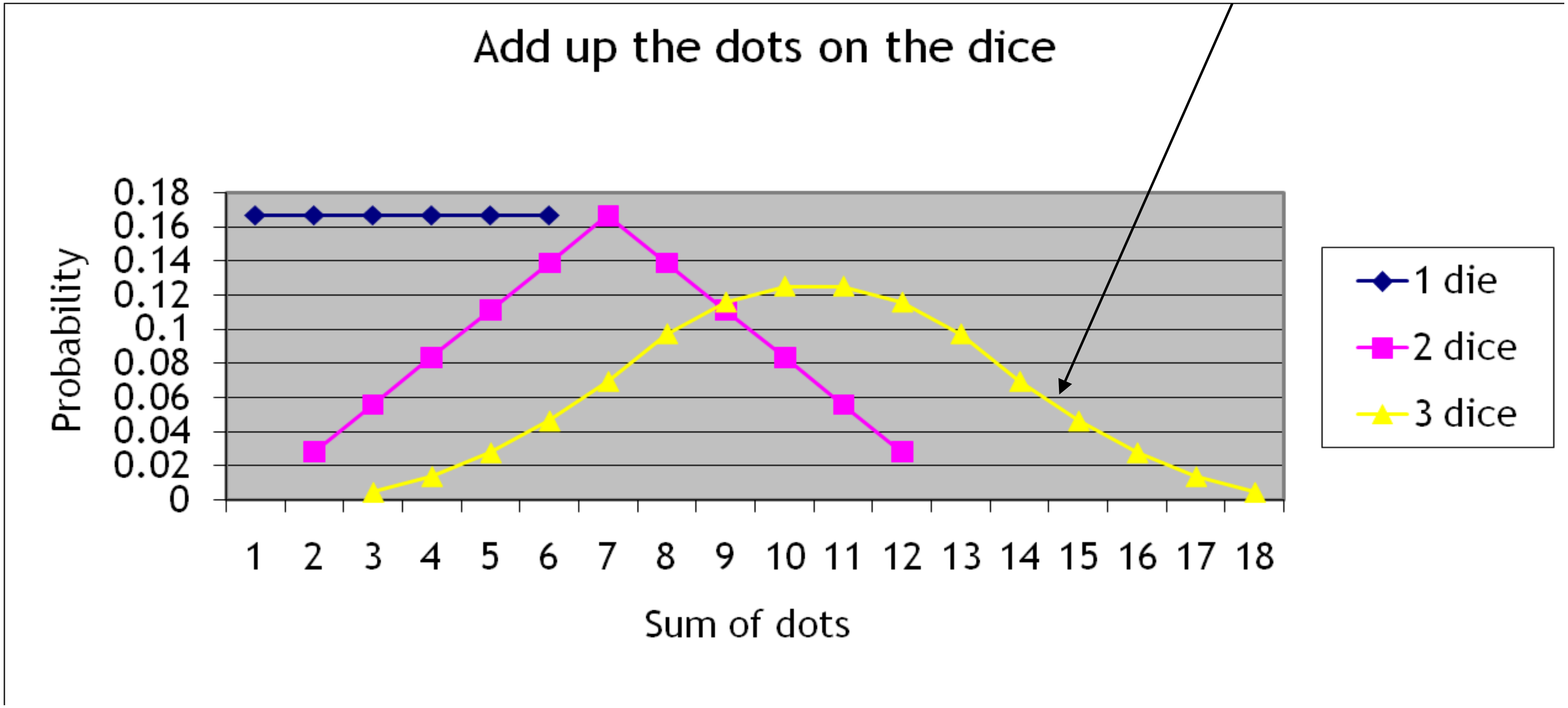
Variability



Variability

Here is why:

Even outcomes that are equally likely (like dice), when you add them up, become bell shaped

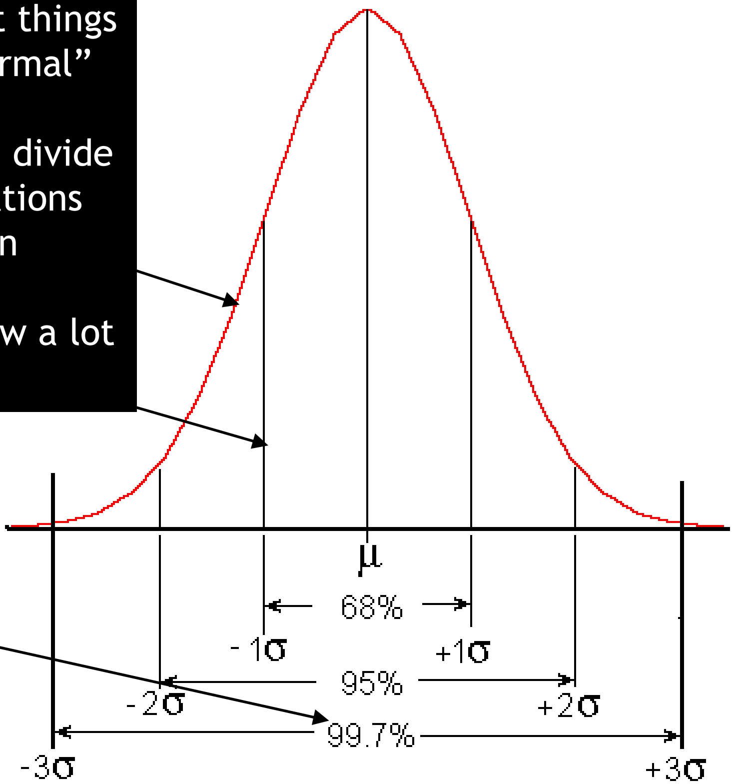


"Normal" bell shaped curve

Add up about 30 of most things and you start to be "normal"

Normal distributions are divide up into 3 standard deviations on each side of the mean

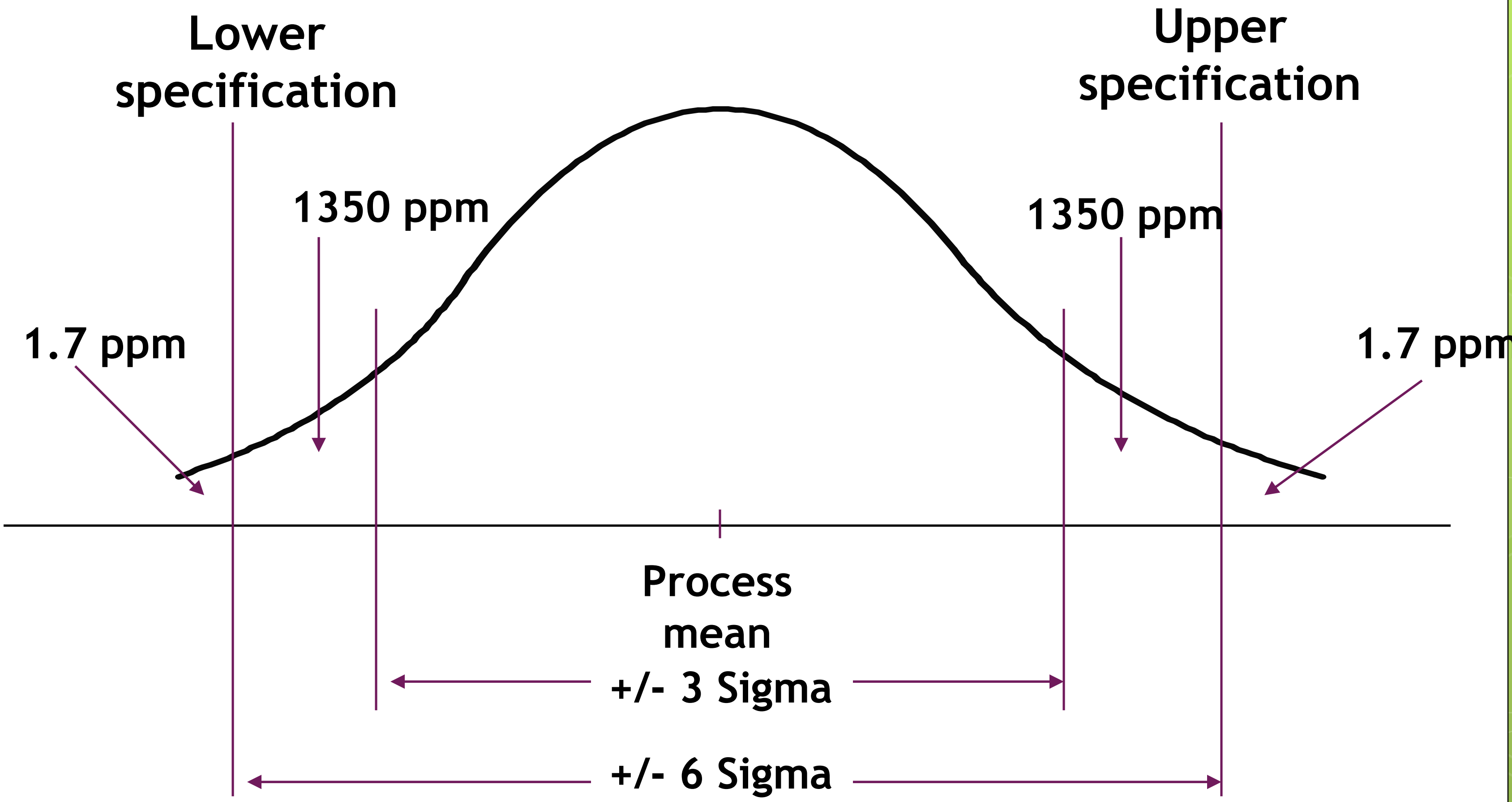
Once your that, you know a lot about what is going on



Causes of Variability

- Common Causes:
  - Random variation (usual)
  - No pattern
  - Inherent in process
  - adjusting the process increases its variation
- Special Causes
  - Non-random variation (unusual)
  - May exhibit a pattern
  - Assignable, explainable, controllable
  - adjusting the process decreases its variation

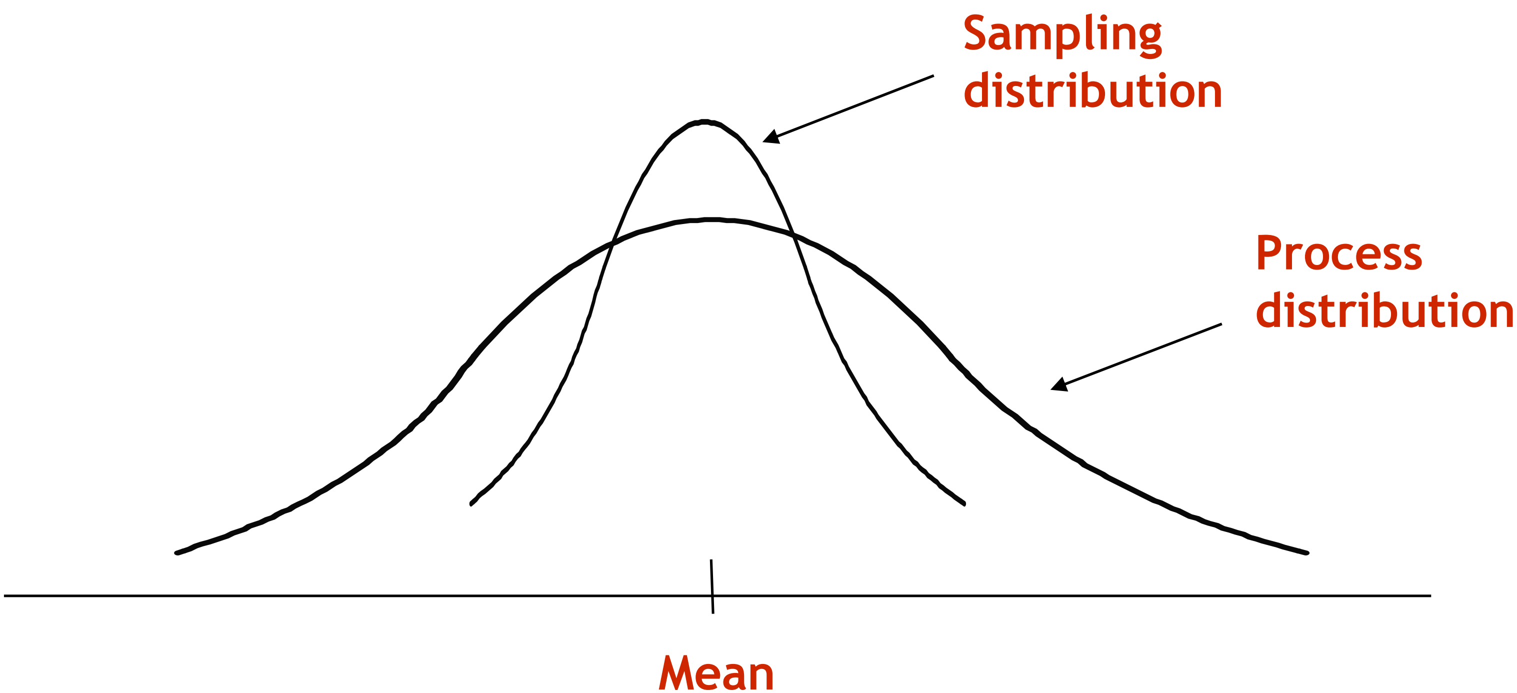
3 Sigma and 6 Sigma Quality



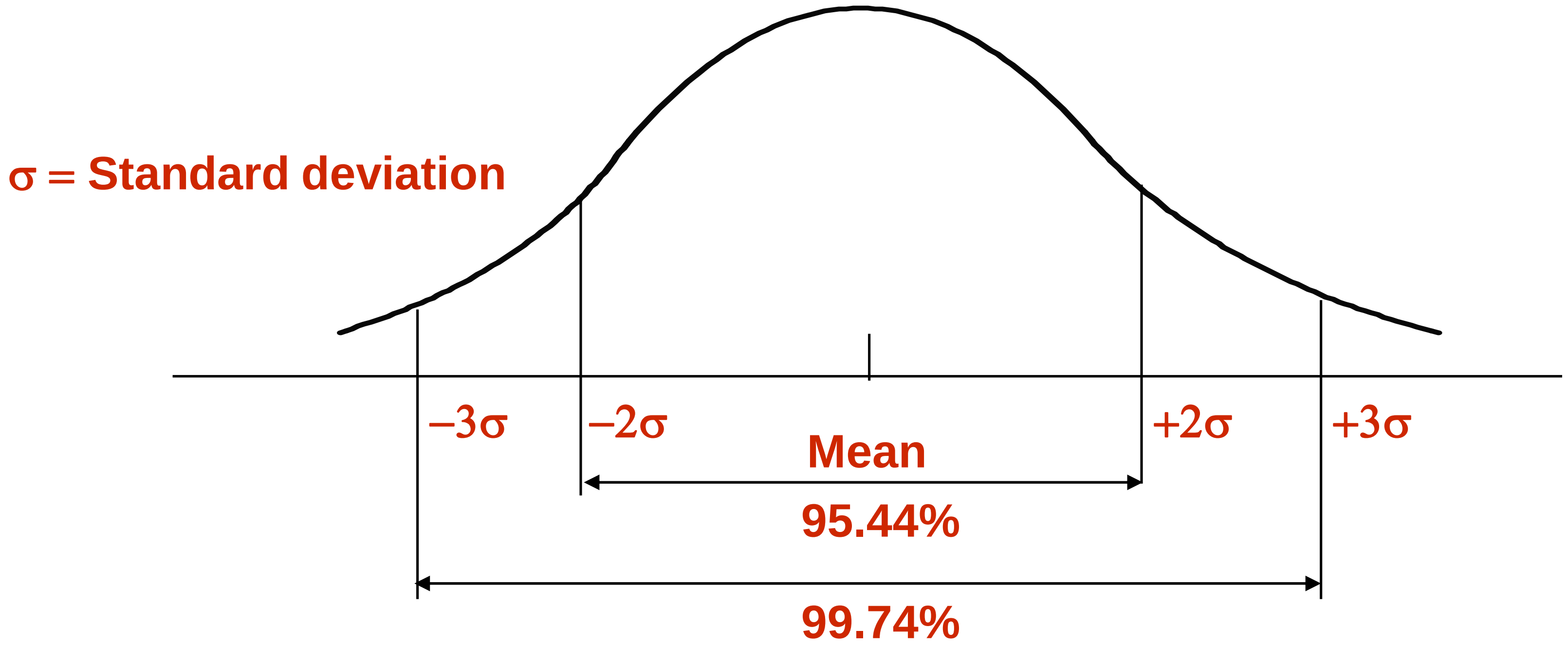
Statistical Process Control

- The Control Process
  - Define
  - Measure
  - Compare
  - Evaluate
  - Correct
  - Monitor results
- Variations and Control
  - Random variation: Natural variations in the output of a process, created by countless minor factors
  - Assignable variation: A variation whose source can be identified

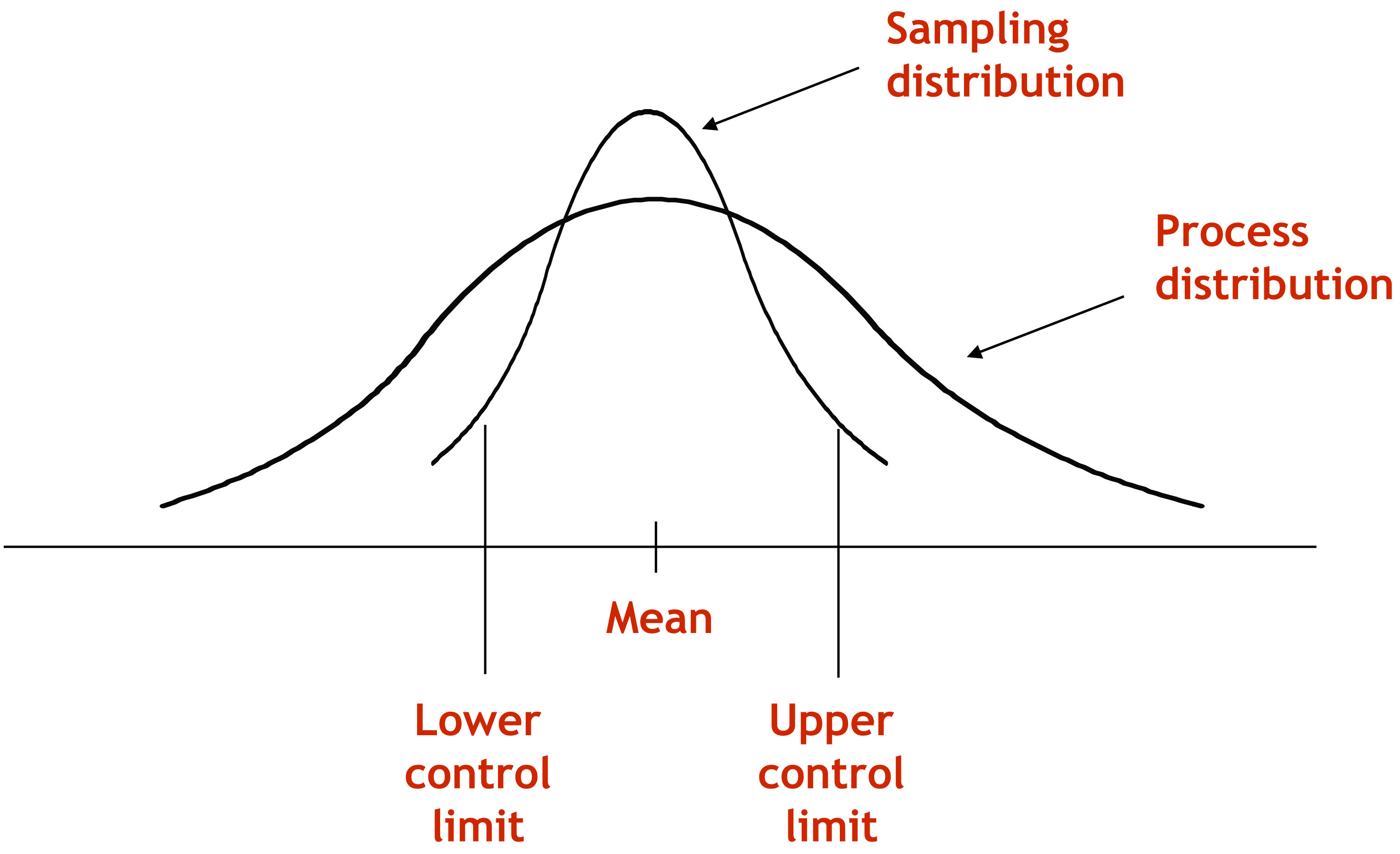
Sampling Distribution



Normal Distribution



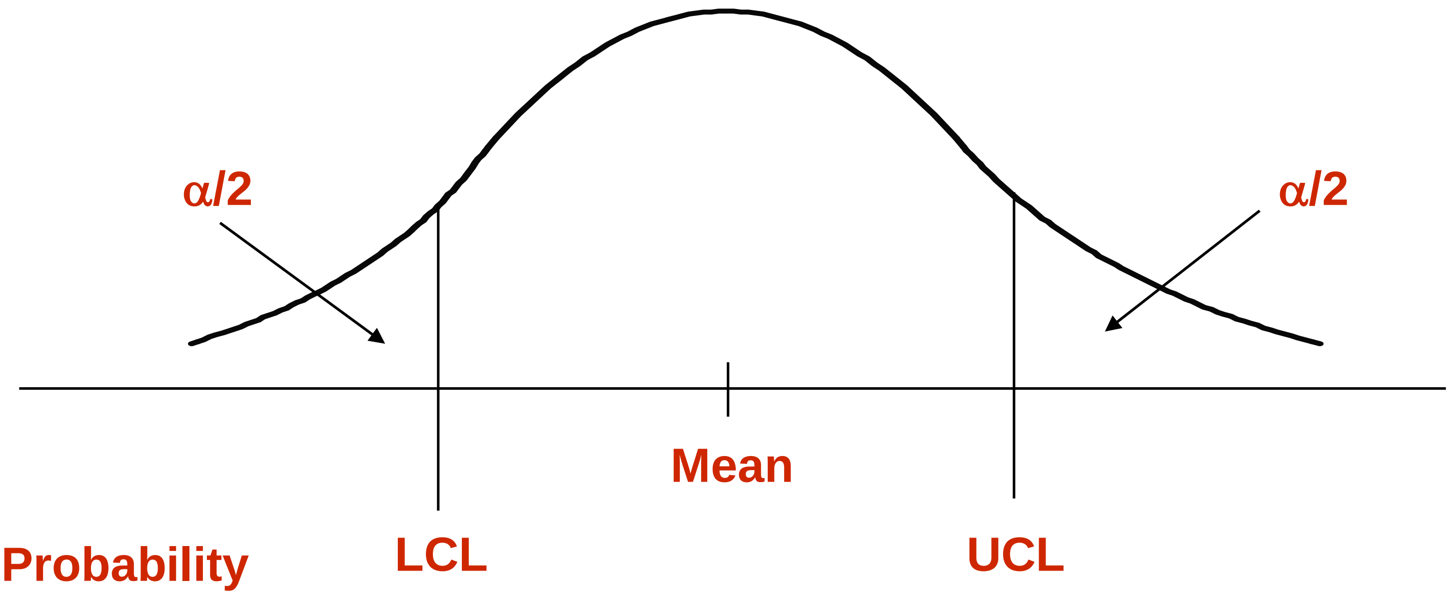
Control Limits



SPC Errors

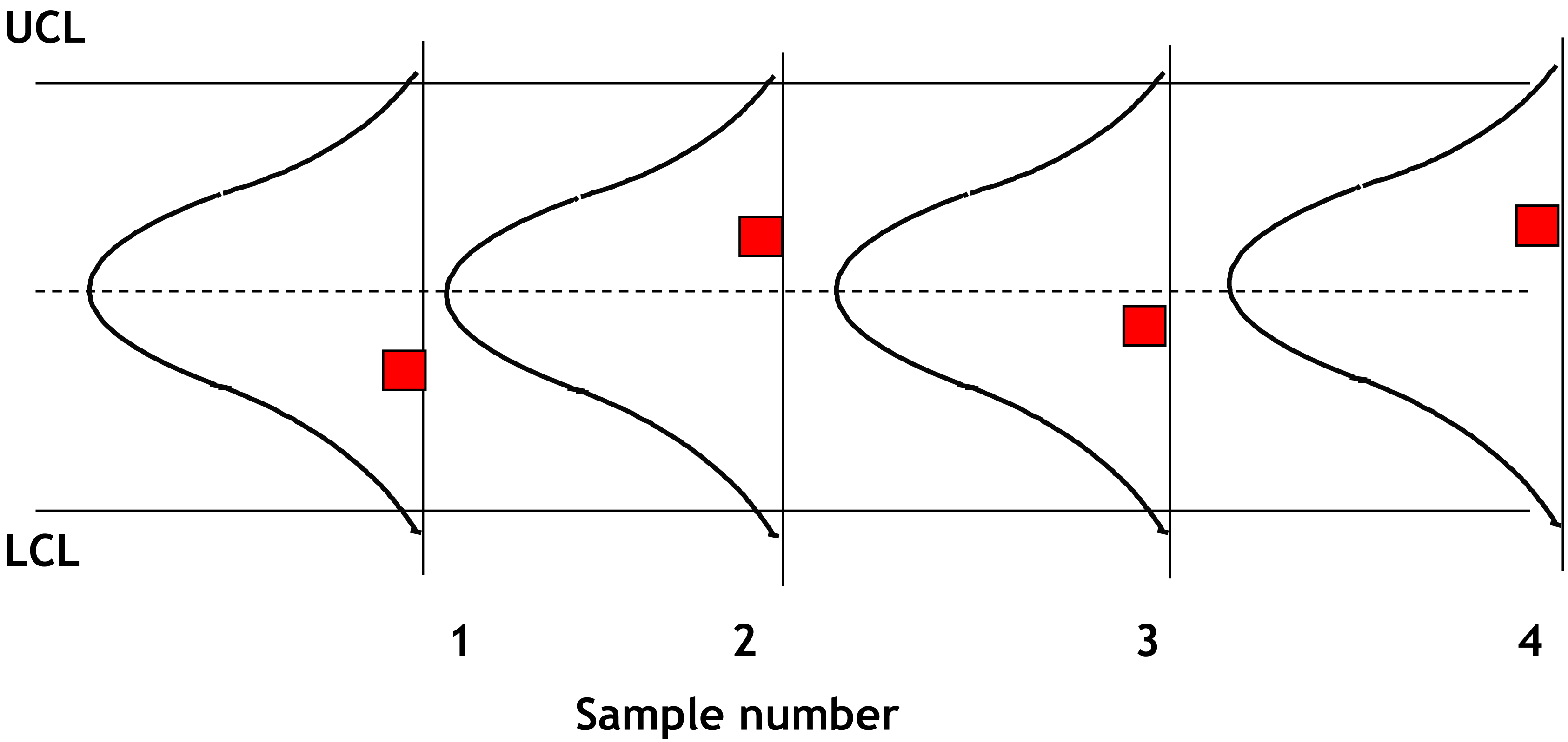
- Type I error
  - Concluding a process is not in control when it actually is.
- Type II error
  - Concluding a process is in control when it is not.

Type I Error



α = Probability of Type I error

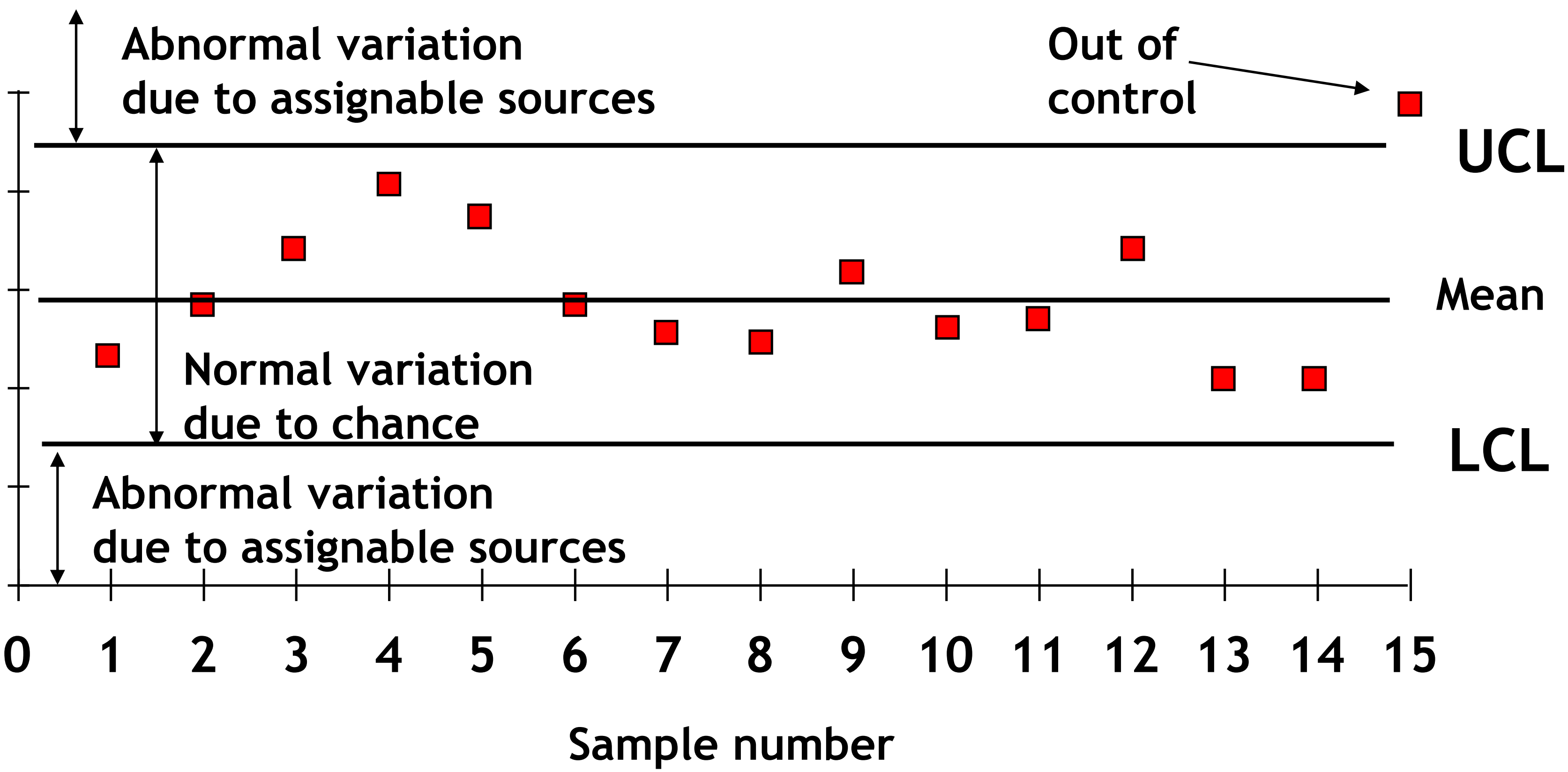
Observations from Sample Distribution



Control Chart

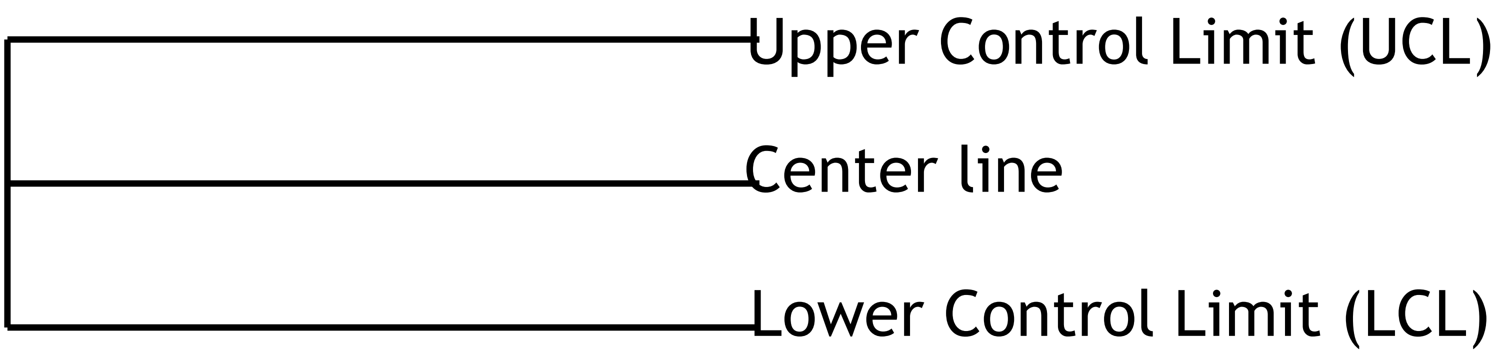
- Control Chart
  - Purpose: to monitor process output to see if it is random
  - A time ordered plot representative sample statistics obtained from an on going process (e.g. sample means)
  - Upper and lower control limits define the range of acceptable variation

Control Chart



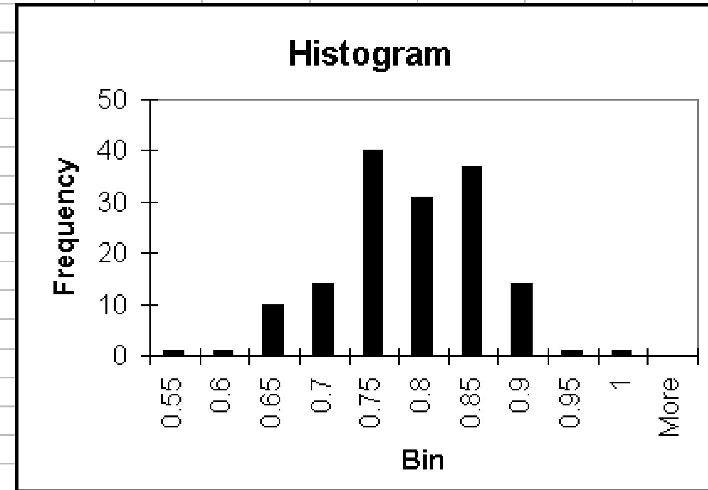
Control Charts in General

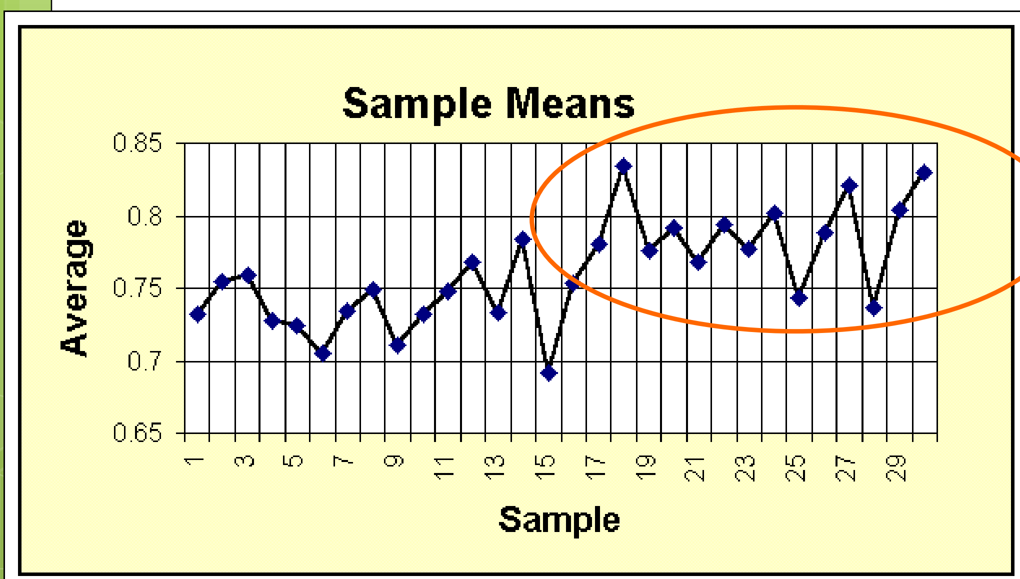
- Are named according to the statistics being plotted, i.e., X bar, R, p, and c
- Have a center line that is the overall average
- Have limits above and below the center line at ± 3 standard deviations (usually)



Histograms do not take into account changes over time.

Frequency distribution	
Upper limit	*Frequency*
0.55	1
0.6	1
0.65	10
0.7	14
0.75	40
0.8	31
0.85	37
0.9	14
0.95	1
1	1
More	0
Average	0.762
Std. Dev.	0.0738





Control charts can tell us when a process changes

Control Chart Applications

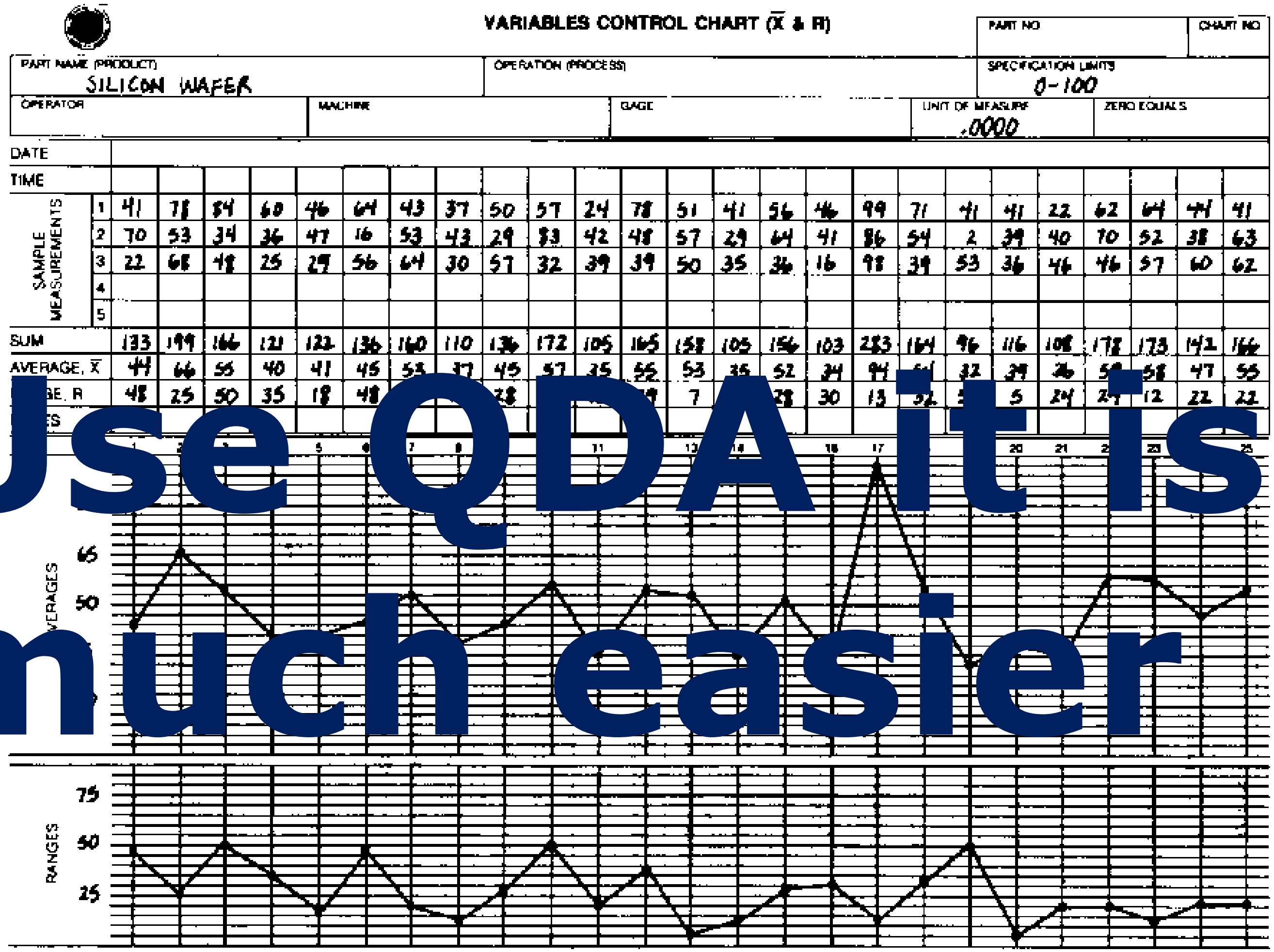
- Establish state of statistical control
- Monitor a process and signal when it goes out of control
- Determine process capability

Commonly Used Control Charts

- Variables data
  - x-bar and R-charts
  - x-bar and s-charts
  - Charts for individuals (x-charts)
- Attribute data
  - For “defectives” (p-chart, np-chart)
  - For “defects” (c-chart, u-chart)

Developing Control Charts

- Prepare
  - Choose measurement
  - Determine how to collect data, sample size, and frequency of sampling
  - Set up an initial control chart
- Collect Data
  - Record data
  - Calculate appropriate statistics
  - Plot statistics on chart



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	41	78	84	60	46	64	43	37	50	57	24	78	51	41	56	46	99	71	41	41	22	62	64	44	41
2	70	53	34	36	47	16	53	43	29	83	42	48	57	29	64	41	86	54	2	39	40	70	52	38	63
3	22	68	48	25	29	56	64	30	57	32	39	39	50	35	36	16	98	39	53	36	46	46	57	60	62
4																									
5																									
SUM	133	199	166	121	122	136	160	110	136	172	105	165	158	105	156	103	283	164	96	116	108	178	173	142	166
AVERAGE, $\bar{X}$	44	66	55	40	41	45	53	37	45	57	35	55	53	35	52	34	94	[illegible]	32	39	36	[illegible]	58	47	55
RANGE, R	48	25	50	35	18	48	[illegible]	[illegible]	28	[illegible]	[illegible]	[illegible]	7	[illegible]	28	30	13	52	[illegible]	5	24	24	12	22	22

Use QDA it is much easier

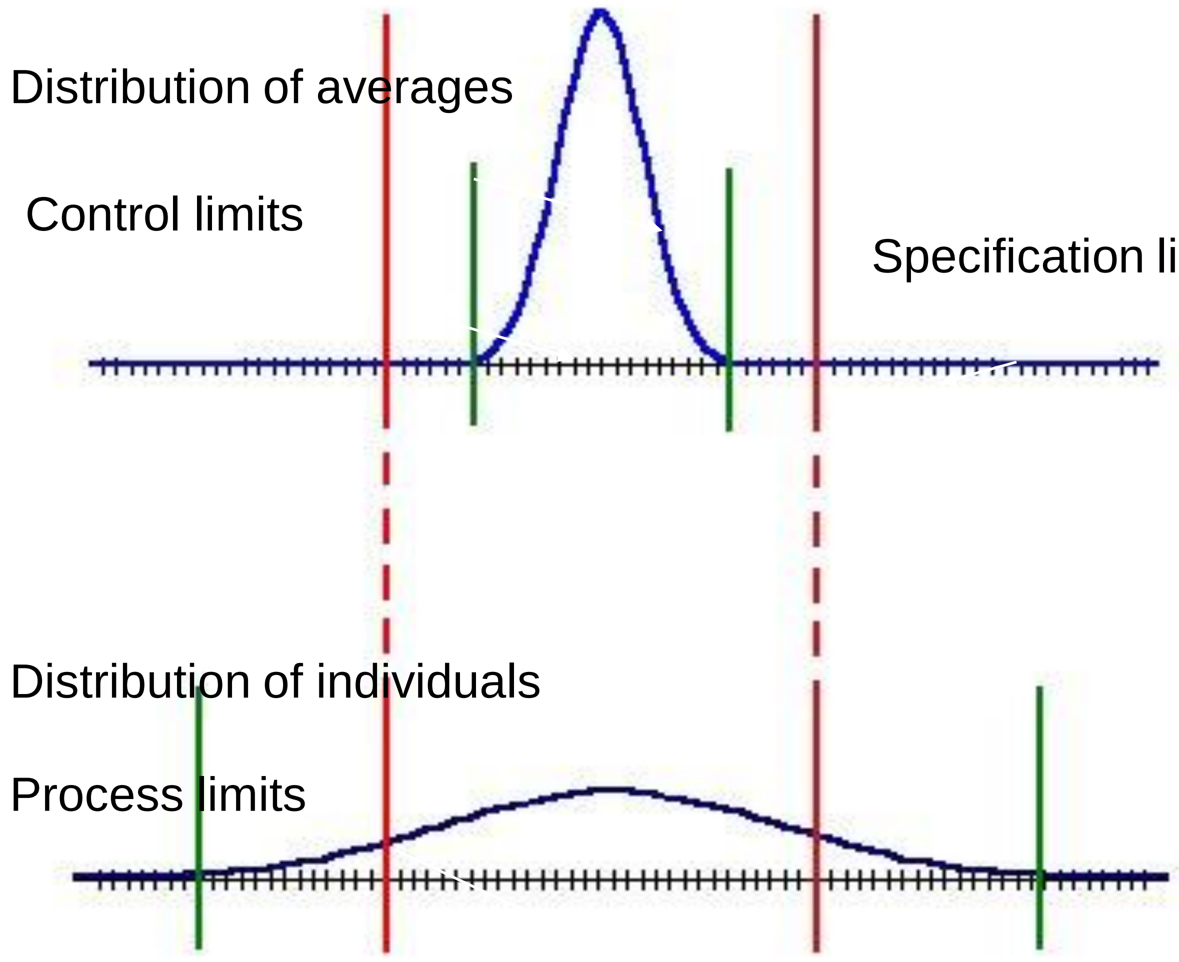
Next Steps

- Determine trial control limits
  - Center line (process average)
  - Compute UCL, LCL
- Analyze and interpret results
  - Determine if in control
  - Eliminate out-of-control points
  - Recompute control limits as necessary

Limits

- Process and Control limits:
  - Statistical
  - Process limits are used for individual items
  - Control limits are used with averages
  - Limits = $\mu \pm 3\sigma$
  - Define usual (common causes) & unusual (special causes)
- Specification limits:
  - Engineered
  - Limits = target ± tolerance
  - Define acceptable & unacceptable

Process vs. control limits



- Variance of averages < variance of individual items

Variables Data Charts

- Process Centering
  - X bar chart
  - X bar is a sample mean

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

- Process Dispersion (consistency)
  - R chart
  - R is a sample range

$$R = \max(X_i) - \min(X_i)$$

X bar charts

- Center line is the grand mean (X double bar)
- Points are X bars

$$\sigma_{\bar{x}} = \sigma / \sqrt{n}$$

$$\bar{\bar{X}} = \frac{\sum_{j=1}^{m} \bar{X}_j}{m}$$

$$UCL = \bar{\bar{X}} + z\sigma_{\bar{x}}$$

$$LCL = \bar{\bar{X}} - z\sigma_{\bar{x}}$$

-OR-

$$UCL = \bar{\bar{X}} + A_2\bar{R}$$

$$LCL = \bar{\bar{X}} - A_2\bar{R}$$

R Charts

- Center line is the grand mean (R bar)
- Points are R
- D3 and D4 values are tabled according to n (sample size)

$$UCL = D_4 \overline{R}$$

$$LCL = D_3 \overline{R}$$

Use of X bar & R charts

- Charts are always used in tandem
- Data are collected (20-25 samples)
- Sample statistics are computed
- All data are plotted on the 2 charts
- Charts are examined for randomness
- If random, then limits are used "forever"

Attribute Charts

- c charts – used to count defects in a constant sample size

$$\bar{c} = \frac{\sum_{i=1}^{n} c}{m} = centerline$$

$$LCL = \bar{c} - z\sqrt{\bar{c}}$$

$$UCL = \bar{c} + z\sqrt{\bar{c}}$$

Attribute Charts

- p charts – used to track a proportion (fraction) defective

$$\bar{p} = \sum_{j=1}^{m} p \Big/ m = \sum_{ij} x \Big/ nm = centerline$$

$$\bar{p} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$UCL = \bar{p} + z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

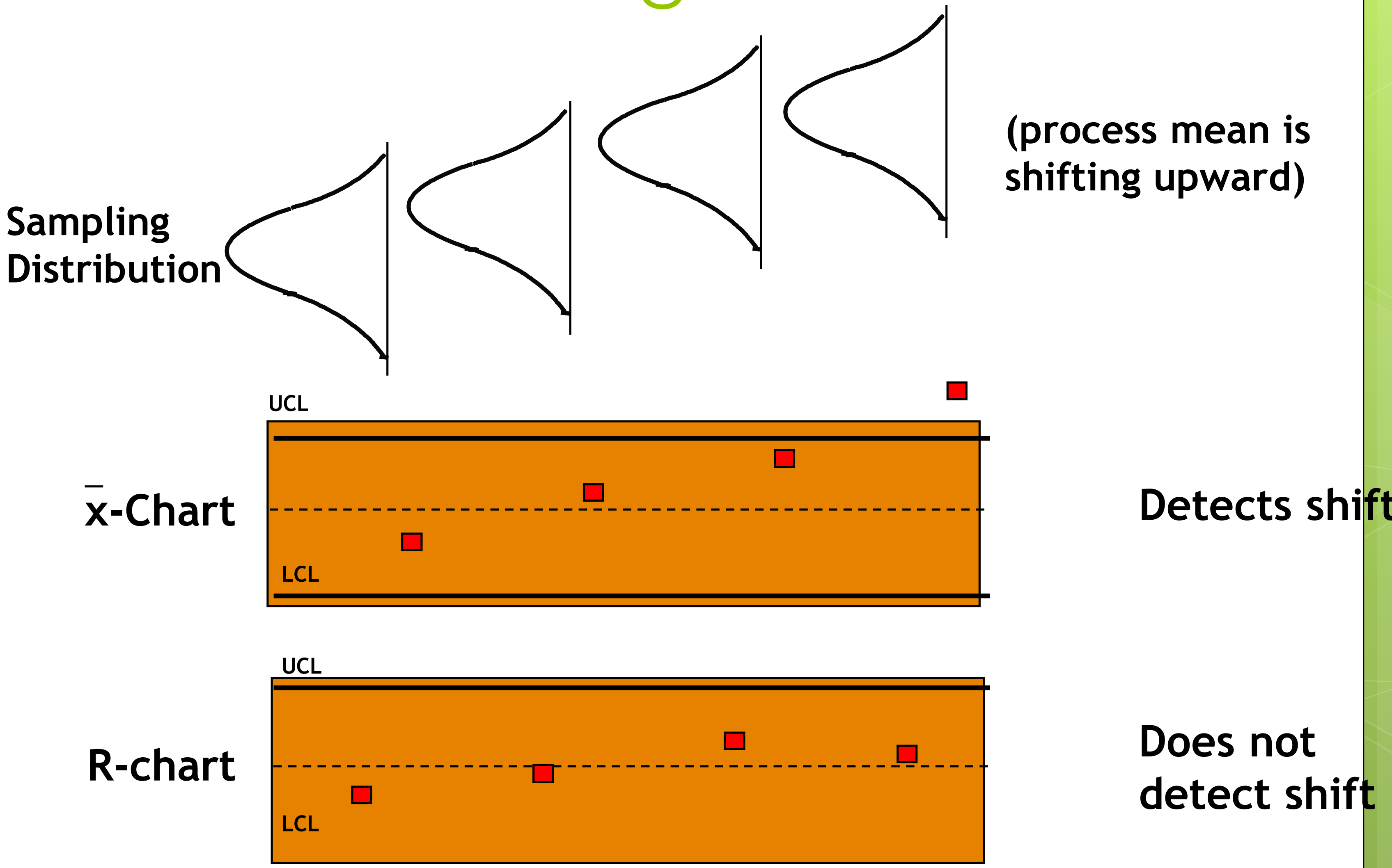
$$LCL = \bar{p} - z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Control Charts for Variables

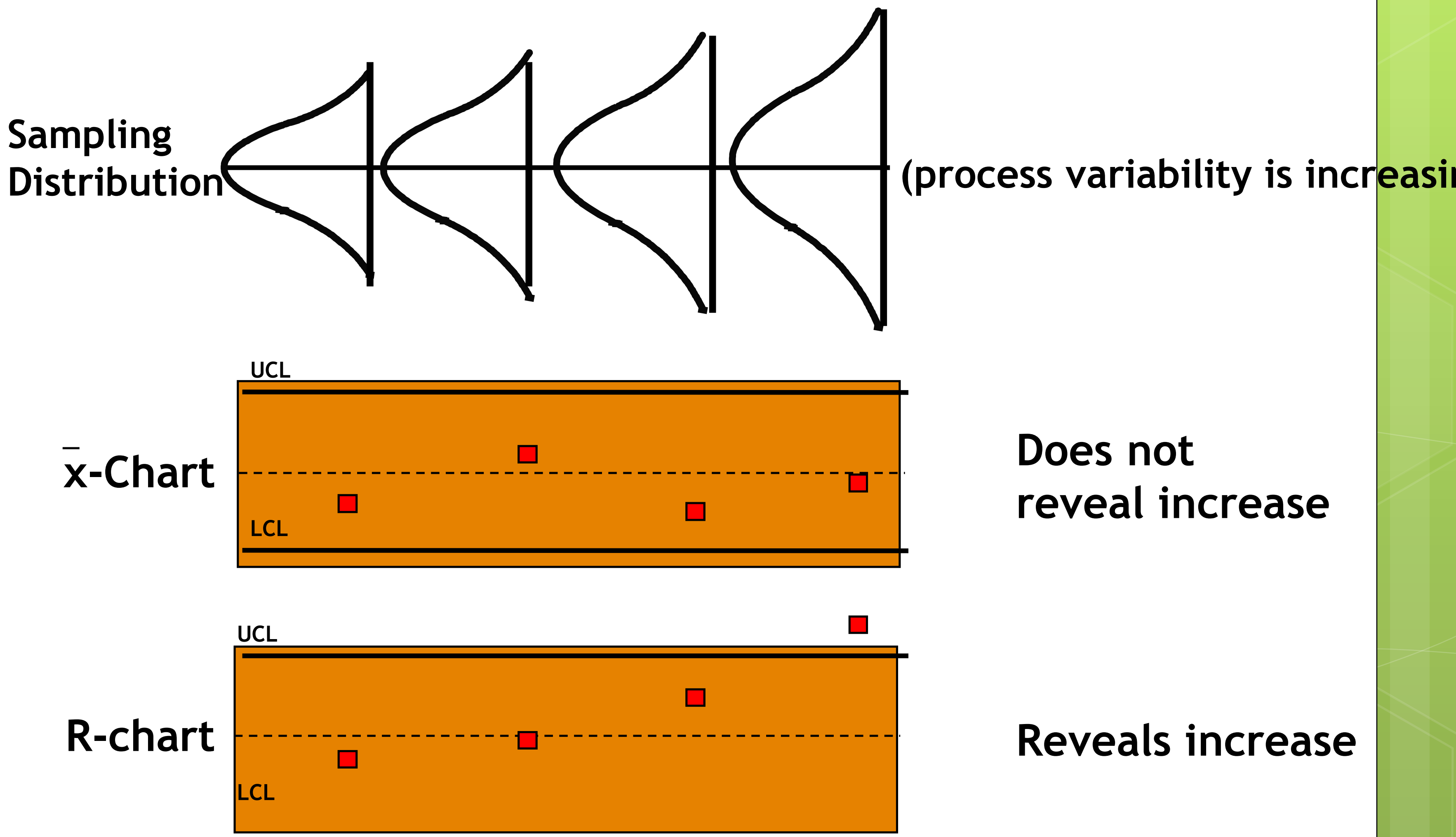
- Mean control charts
  - Used to monitor the central tendency of a process.
  - X bar charts
- Range control charts
  - Used to monitor the process dispersion
  - R charts

Variables generate data that are *measured*.

Mean and Range Charts



Mean and Range Charts



Control Chart for Attributes

- p-Chart - Control chart used to monitor the proportion of defectives in a process
- c-Chart - Control chart used to monitor the number of defects per unit

Attributes generate data that are *counted*.

Use of p-Charts

- When observations can be placed into two categories.
  - Good or bad
  - Pass or fail
  - Operate or don't operate
- When the data consists of multiple samples of several observations each

Use of c-Charts

- Use only when the number of occurrences per unit of measure can be counted; non-occurrences cannot be counted.
  - Scratches, chips, dents, or errors per item
  - Cracks or faults per unit of distance
  - Breaks or Tears per unit of area
  - Bacteria or pollutants per unit of volume
  - Calls, complaints, failures per unit of time

Use of Control Charts

- At what point in the process to use control charts
- What size samples to take
- What type of control chart to use
  - Variables
  - Attributes

Run Tests

- Run test – a test for randomness
- Any sort of pattern in the data would suggest a non-random process
- All points are within the control limits - the process may not be random

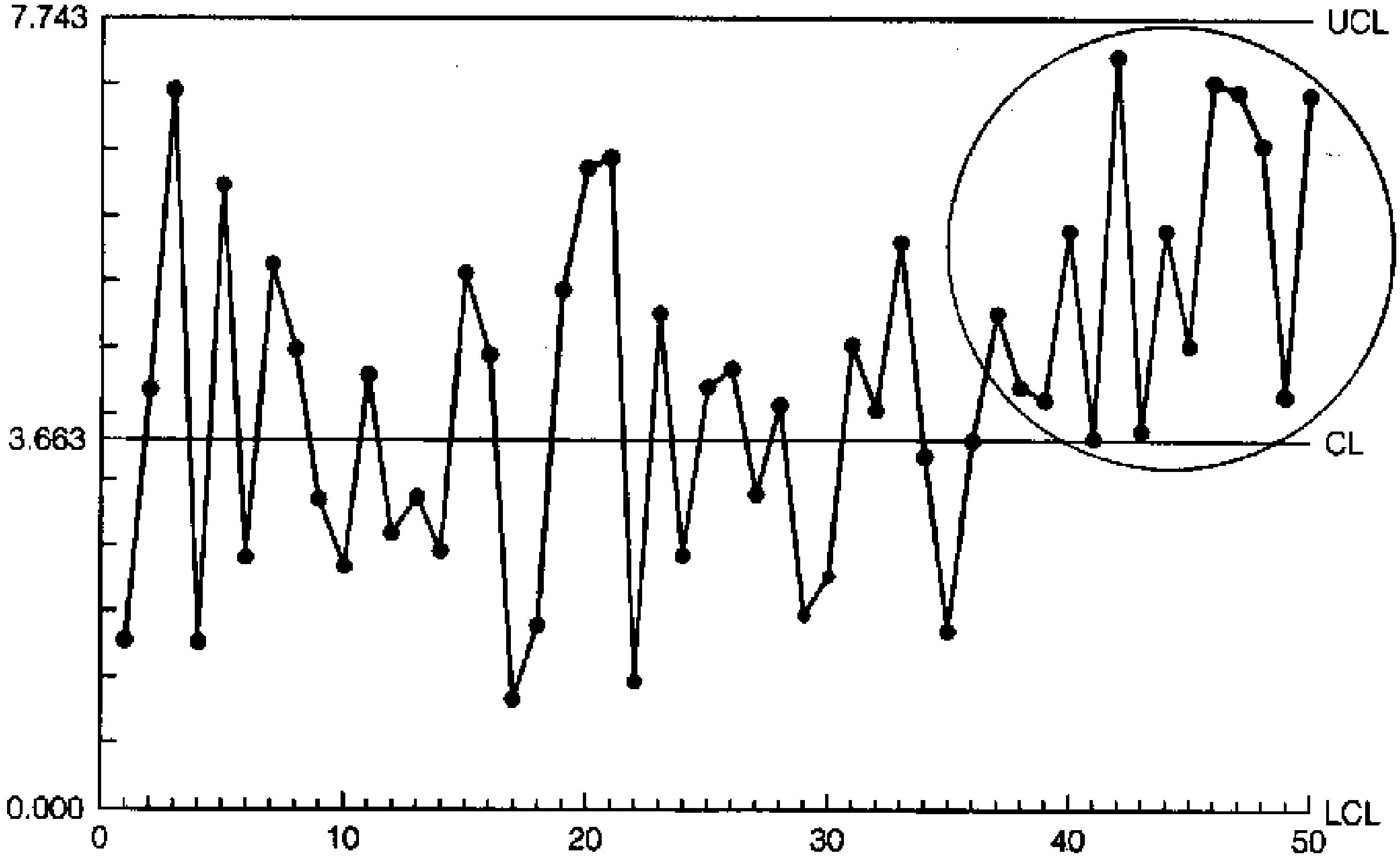
Nonrandom Patterns in Control charts

- Trend
- Cycles
- Bias
- Mean shift
- Too much dispersion

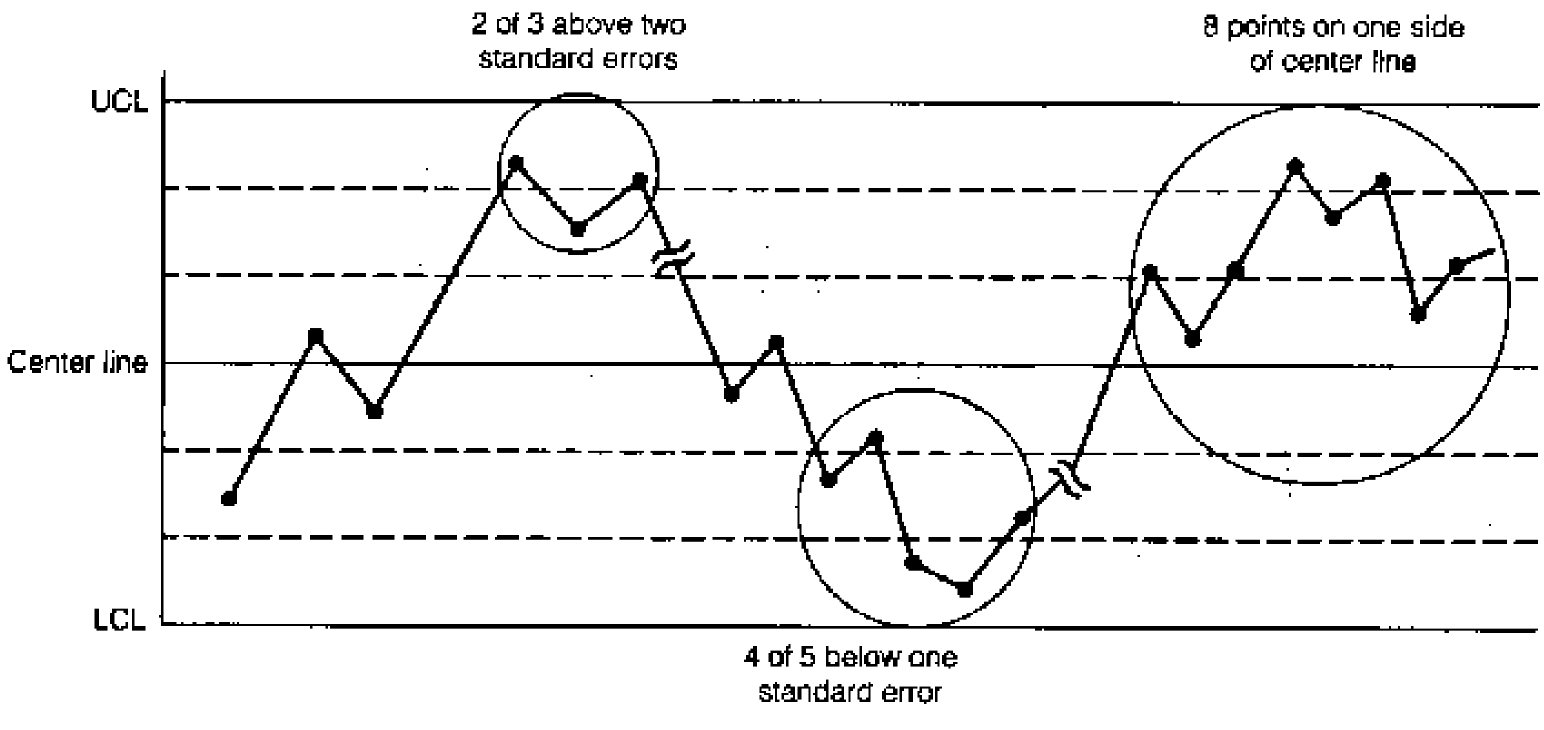
Typical Out-of-Control Patterns

- Point outside control limits
- Sudden shift in process average
- Cycles
- Trends
- Hugging the center line
- Hugging the control limits
- Instability

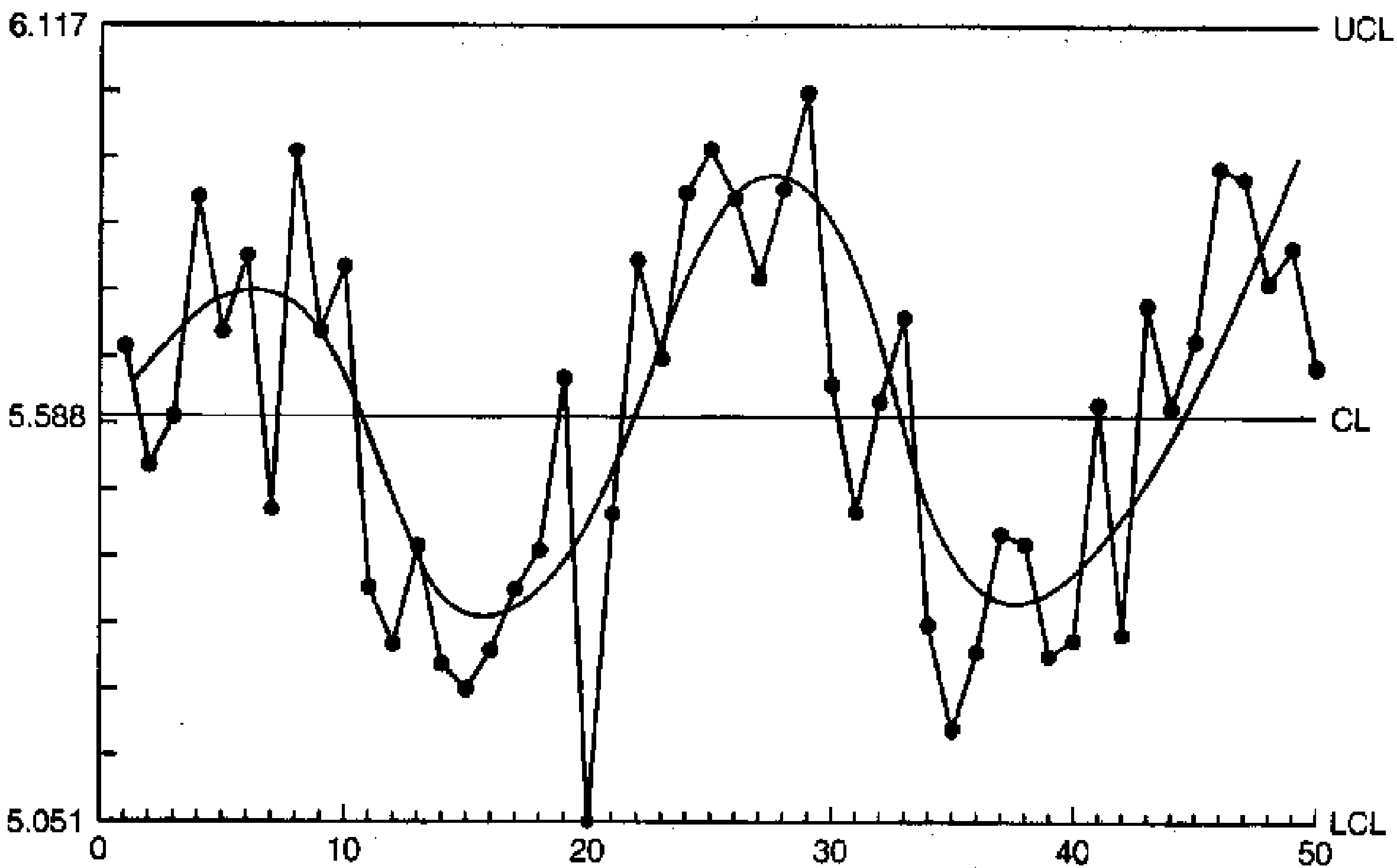
Shift in Process Average



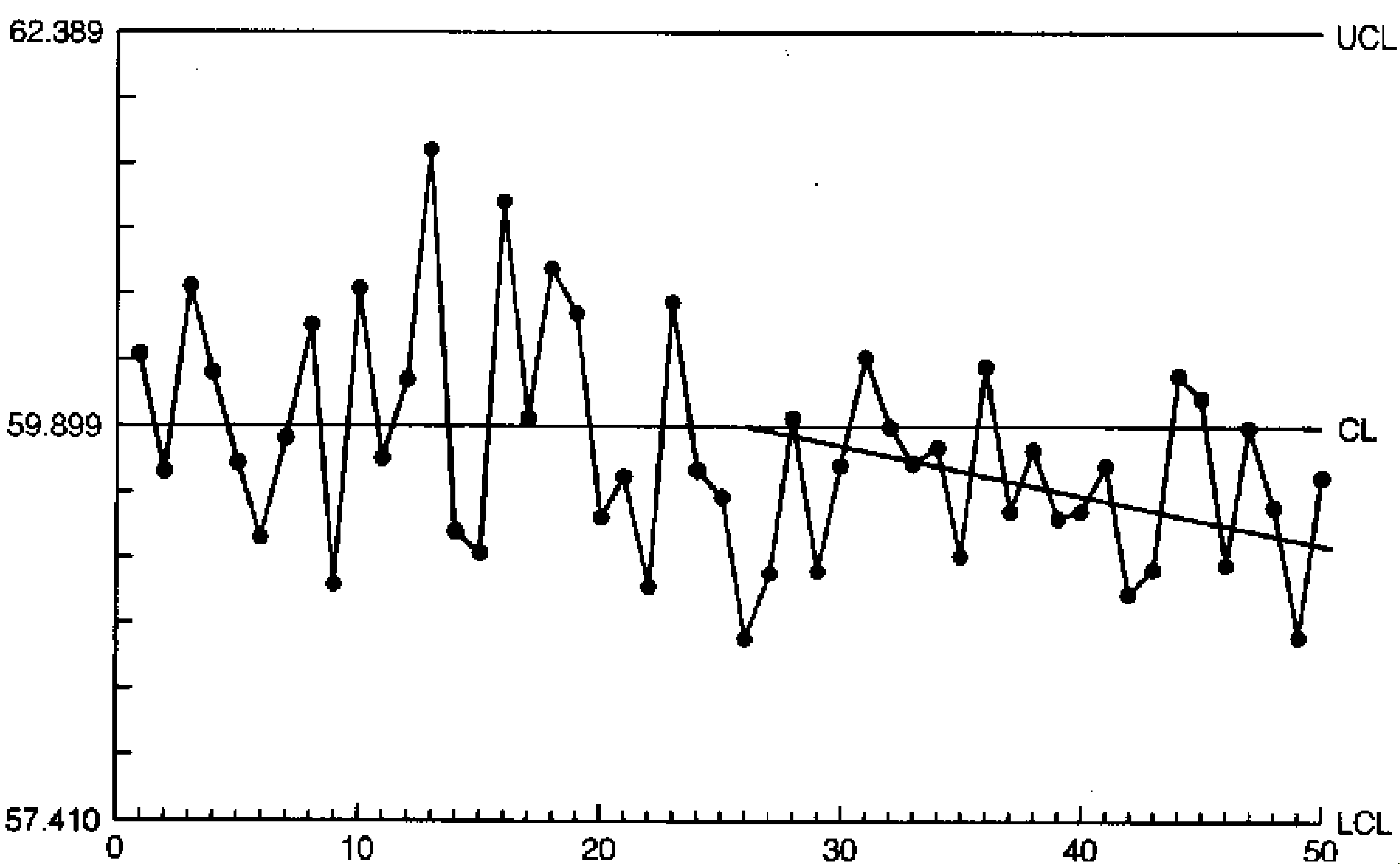
Identifying Potential Shifts



Cycles



Trend



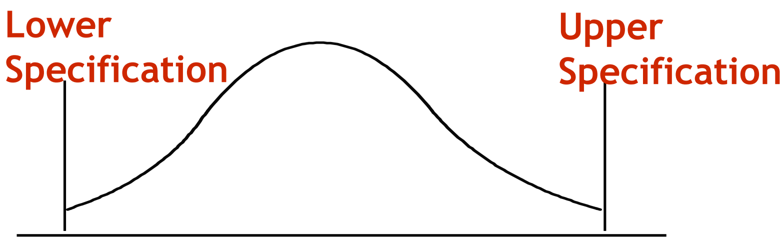
Final Steps

- Use as a problem-solving tool
  - Continue to collect and plot data
  - Take corrective action when necessary
- Compute process capability

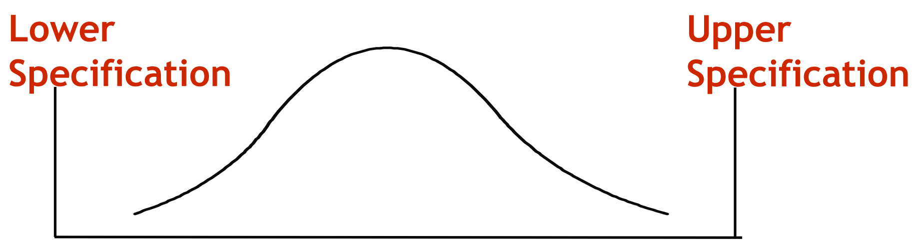
Process Capability

- Tolerances or specifications
  - Range of acceptable values established by engineering design or customer requirements
- Process variability
  - Natural variability in a process
- Process capability
  - Process variability relative to specification

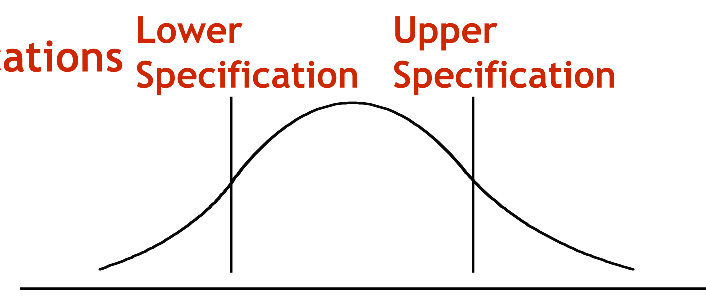
Process Capability



A. Process variability matches specifications



B. Process variability well within specifications



C. Process variability exceeds specifications

Process Capability Ratio

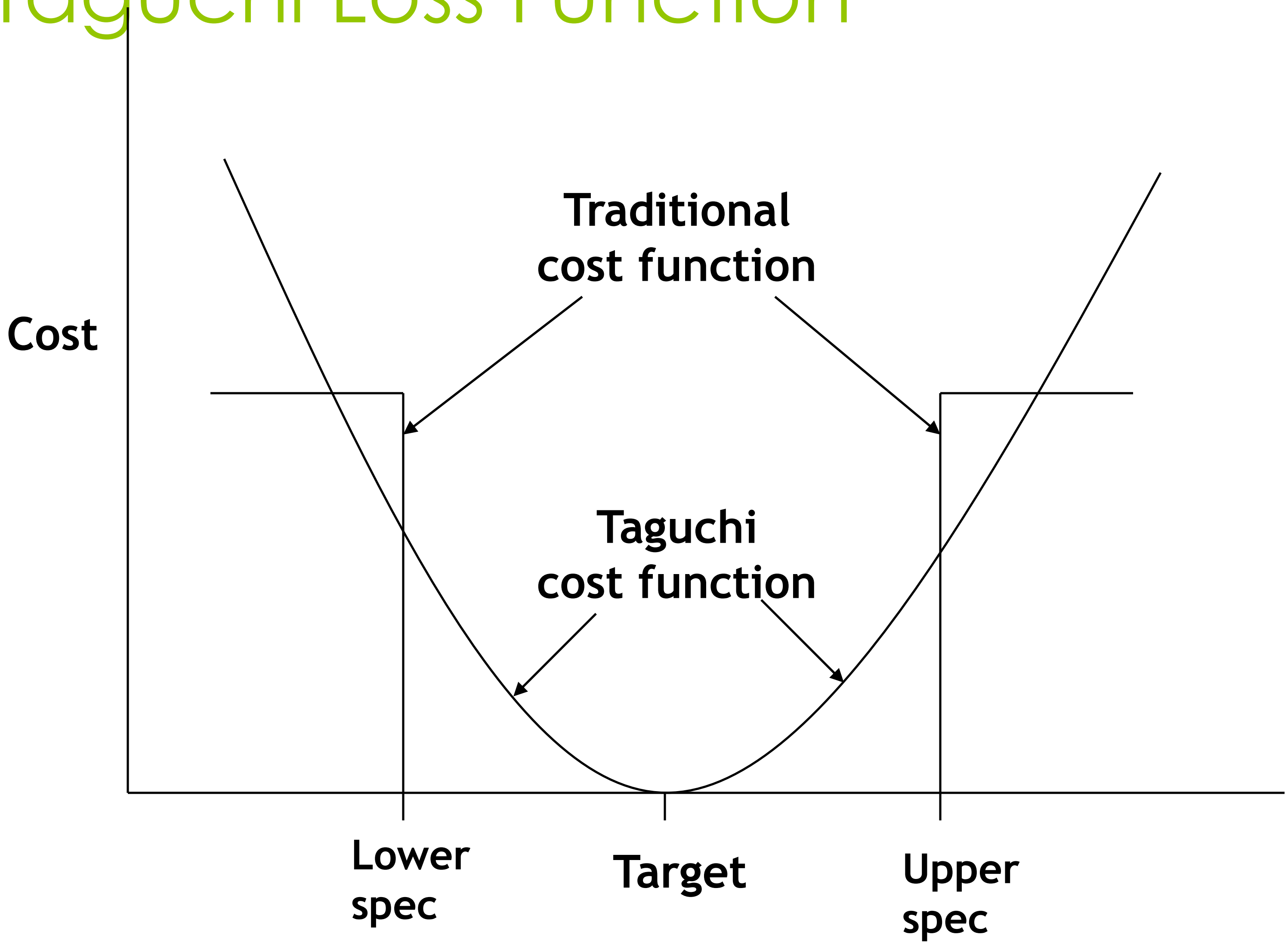
$$\text{Process capability ratio, Cp} = \frac{\text{specification width}}{\text{process width}}$$

$$\text{Cp} = \frac{\text{Upper specification – lower specification}}{6\sigma}$$

Improving Process Capability

- Simplify
- Standardize
- Mistake-proof
- Upgrade equipment
- Automate

Taguchi Loss Function



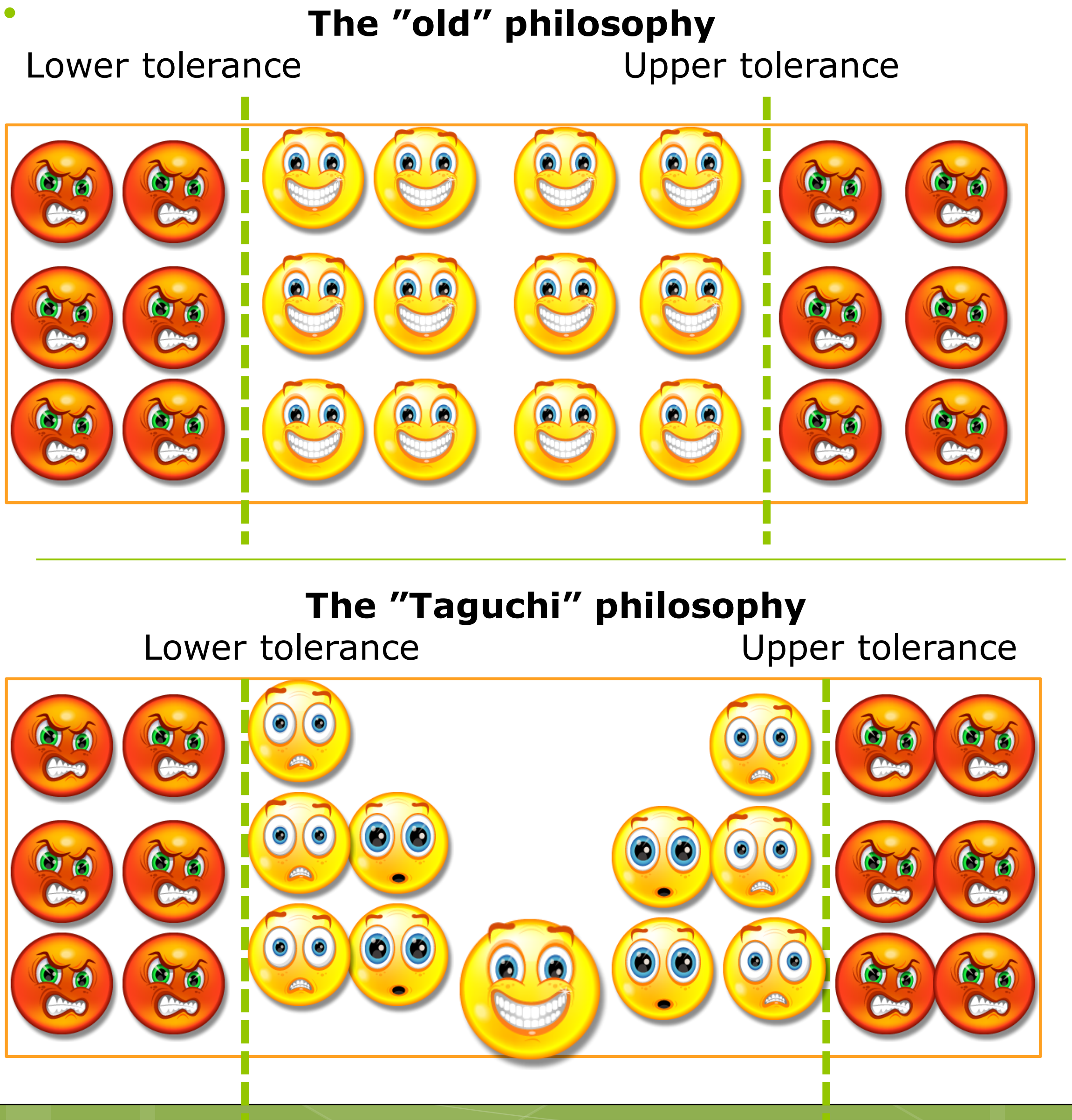
Meet the Guru: Taguchi

Taguchi

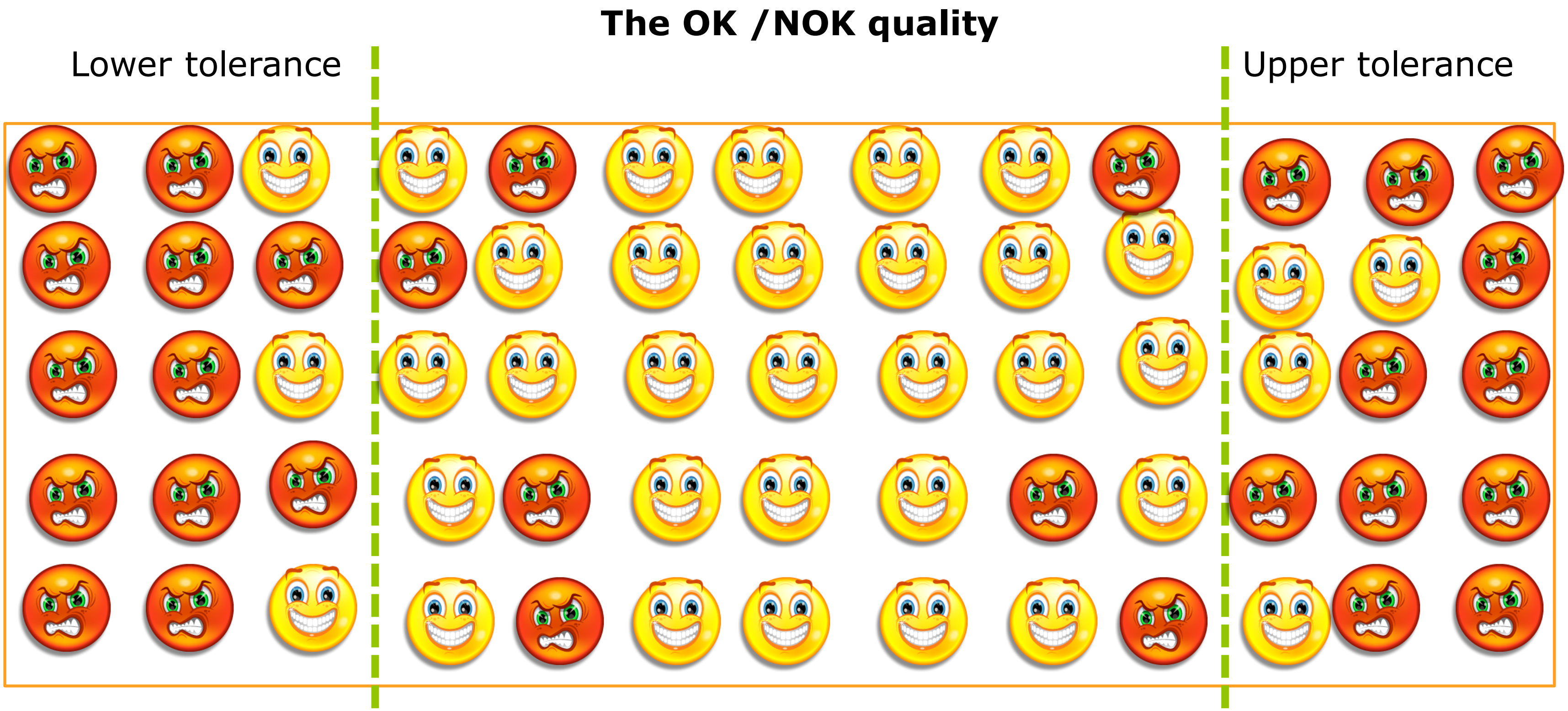
Quality through design

1) Product must bee robust to variation in process

2) Loss is equal to distance from nominal



But in reality it looks like this ?

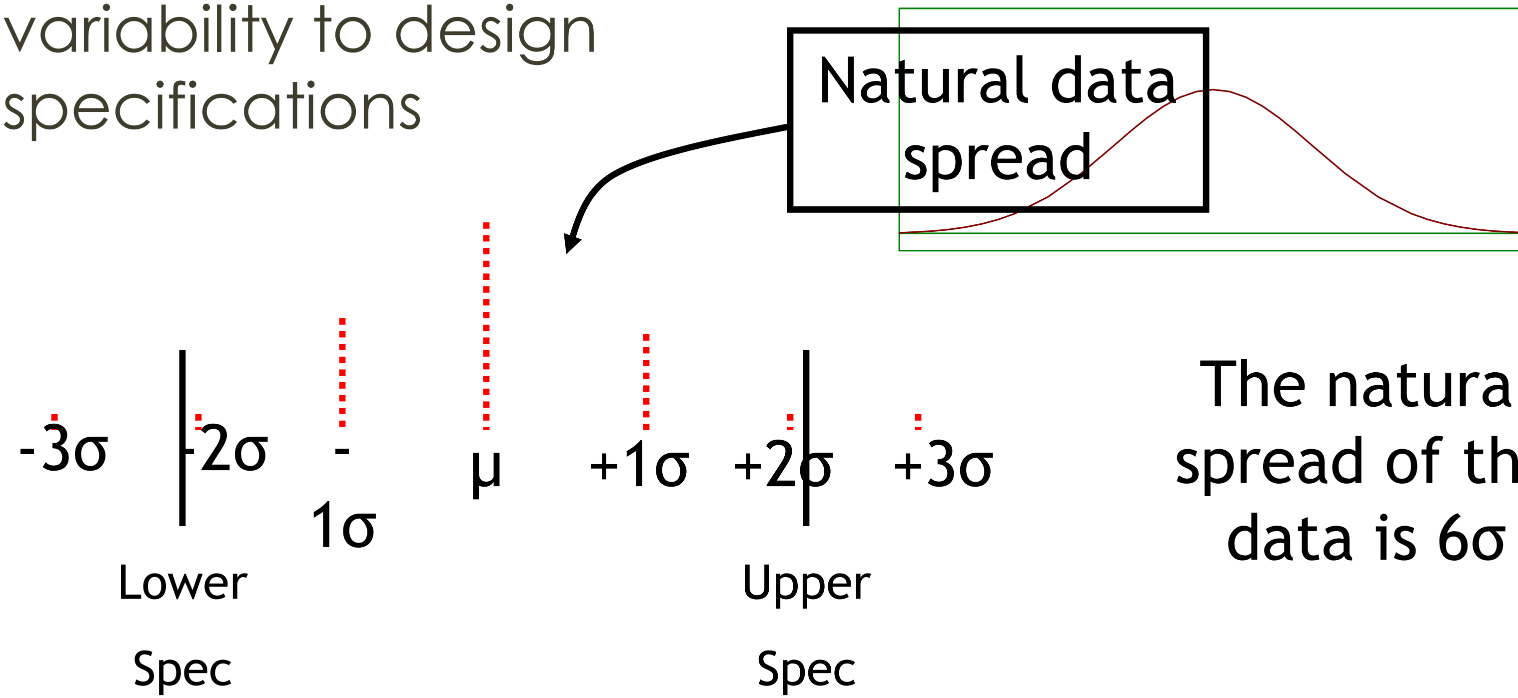


Limitations of Capability Indexes

- Process may not be stable
- Process output may not be normally distributed
- Process not centered but Cp is used

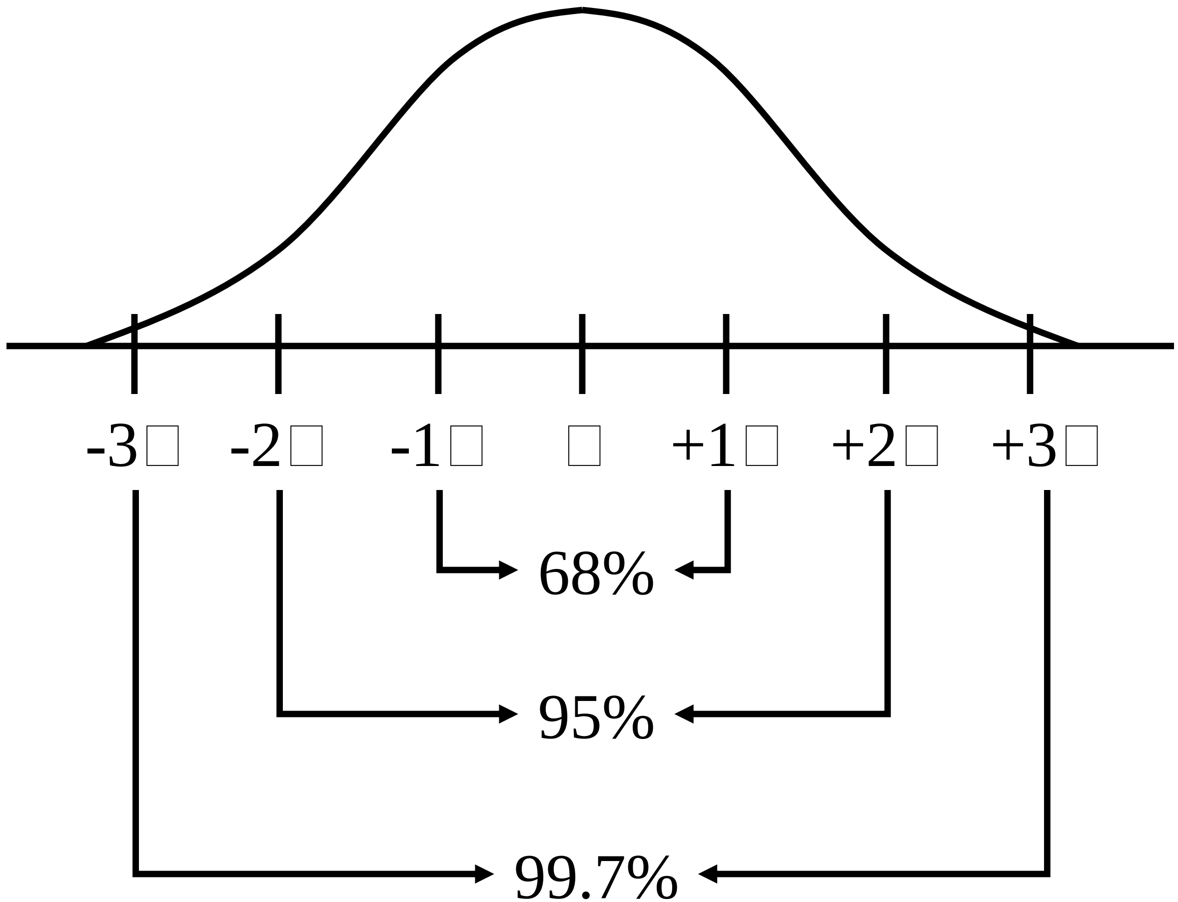
Process Capability

- The ratio of process variability to design specifications



The natural spread of the data is 6σ

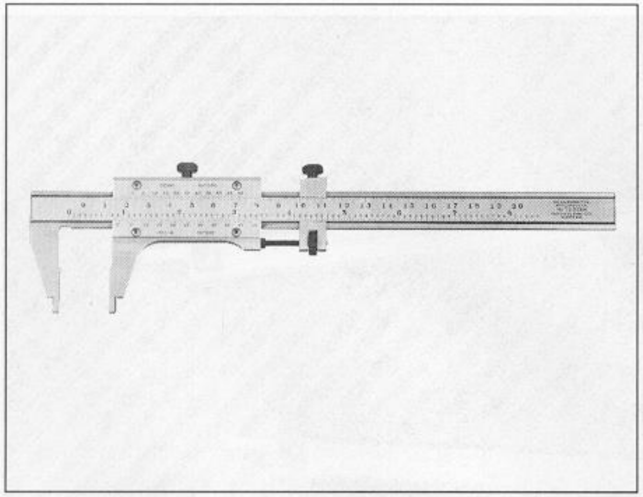
Empirical Rule



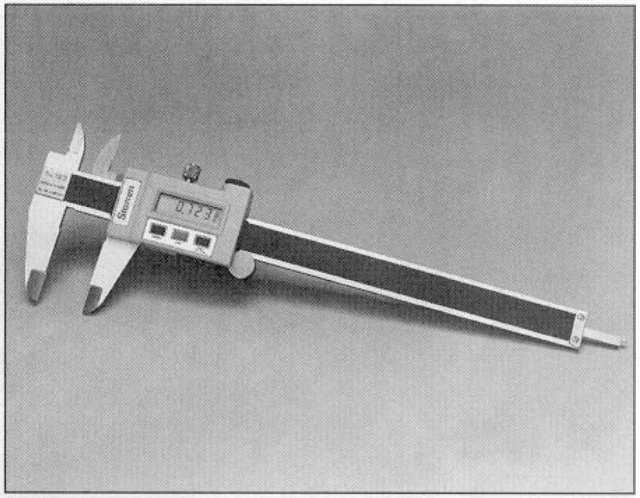
Gauges and Measuring Instruments

- Variable gauges
- Fixed gauges
- Coordinate measuring machine
- Vision systems

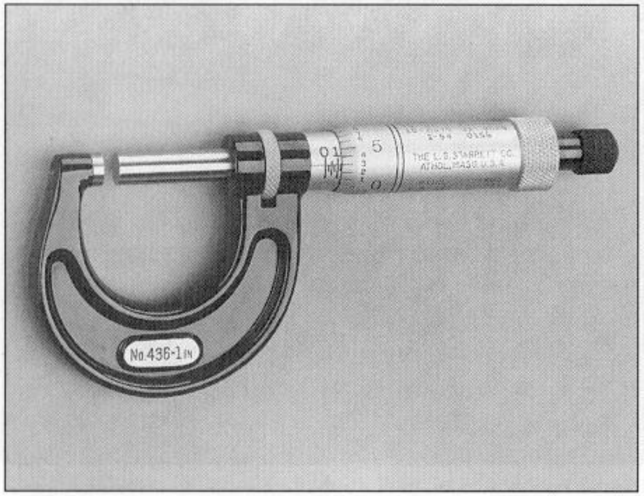
Examples of Gauges



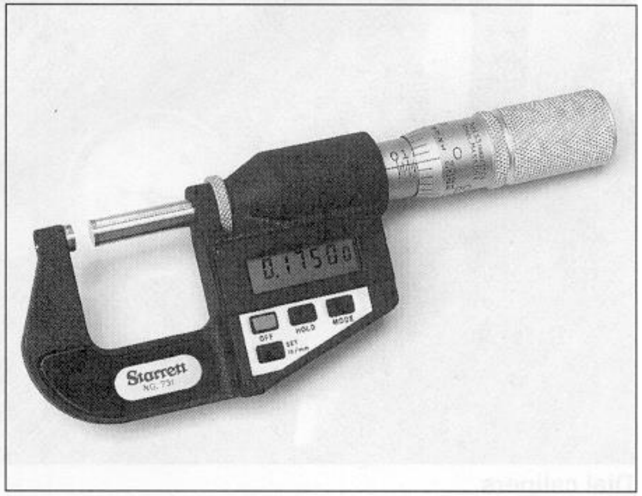
Vernier caliper



Digital caliper



Micrometer



One-inch digital electronic micrometer

Metrology - Science of Measurement

- Accuracy - closeness of agreement between an observed value and a standard
- Precision - closeness of agreement between randomly selected individual measurements

Repeatability and Reproducibility

- Repeatability (equipment variation) – variation in multiple measurements by an individual using the same instrument.
- Reproducibility (operator variation) - variation in the same measuring instrument used by different individuals

Repeatability and Reproducibility Studies

- Quantify and evaluate the capability of a measurement system
  - Select m operators and n parts
  - Calibrate the measuring instrument
  - Randomly measure each part by each operator for r trials
  - Compute key statistics to quantify repeatability and reproducibility

Reliability and Reproducibility Studies(2)

Measurement (M) made by
Operators (i from 1 to m) on
Parts (j from 1 to n) in
Trials (k from 1 to r)

Use QDA it is much easier

$$\bar{x}_i = \frac{\left(\sum_j \sum_k M_{ijk}\right)}{n \cdot r} \text{ average for each operator}$$

$$\bar{x}_D = \max_i(\bar{x}_i) - \min_i(\bar{x}_i) \text{ difference (range) of operator averages}$$

$$R_{ij} = \max_k(M_{ijk}) - \min_k(M_{ijk}) \text{ range for each part for each operator}$$

$$\bar{R}_i = \frac{\left(\sum_j R_{ij}\right)}{n} \text{ average range for each operator}$$

$$\bar{\bar{R}} = \frac{\left(\sum_i \bar{R}_i\right)}{m} \text{ average range of all}$$

Reliability and Reproducibility Studies(3)

Control limit of ranges $R_{ij} = D_4 \cdot \overline{\overline{R}}$

Use number trials (r) for n in table. Check for randomness of errors.

Repeatability or Equipment Variation

$EV = K_1 \cdot \overline{\overline{R}}$ K_1 is a constant tied to # of trials

Reproducibility or operator appraisal variation

$$AV = \sqrt{(K_2 \cdot \bar{x}_D)^2 - \left(\frac{EV^2}{n \cdot r}\right)}$$ K_2 is a constant tied to # of operators

Repeatability and Reproducibility

$$R\&R = \sqrt{(EV)^2 + (AV)^2}$$

Use QDA it is much easier

Results are in actual units measured. Customary to express as percentages.

Under 10% - Acceptable

10 - 30% - ? based on importance and repair cost

Over 30% - Unacceptable

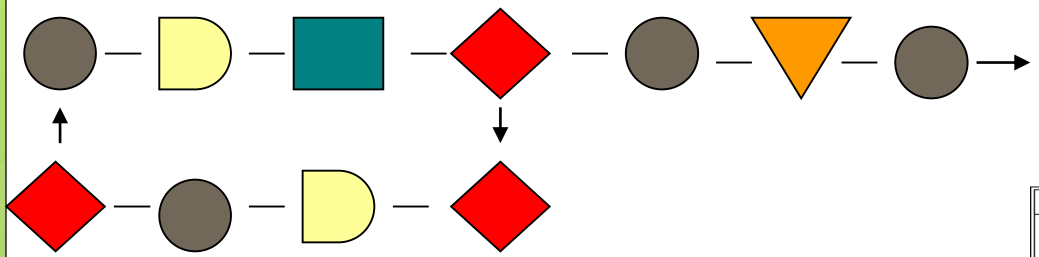
R&R Constants

Number of Trials	2	3	4	5
K_1	4.56	3.05	2.50	2.21
Number of Operators	2	3	4	5
K_2	3.65	2.70	2.30	2.08

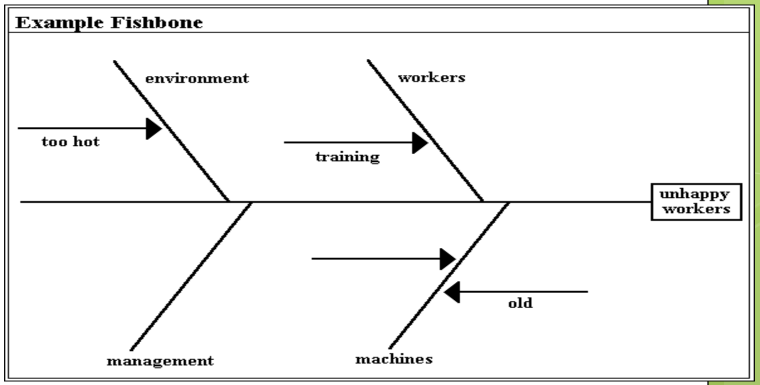
R&R Evaluation

- Under 10% error - OK
- 10-30% error - may be OK
- over 30% error - unacceptable

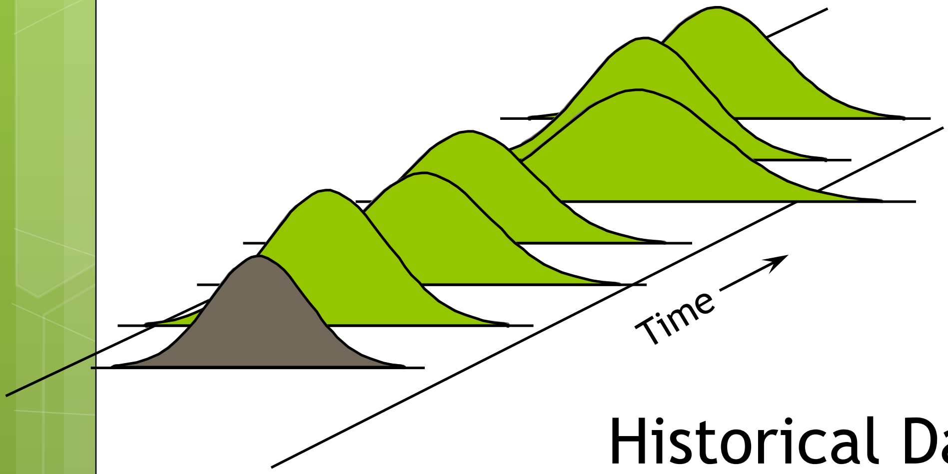
How do we know our process?



Process Map

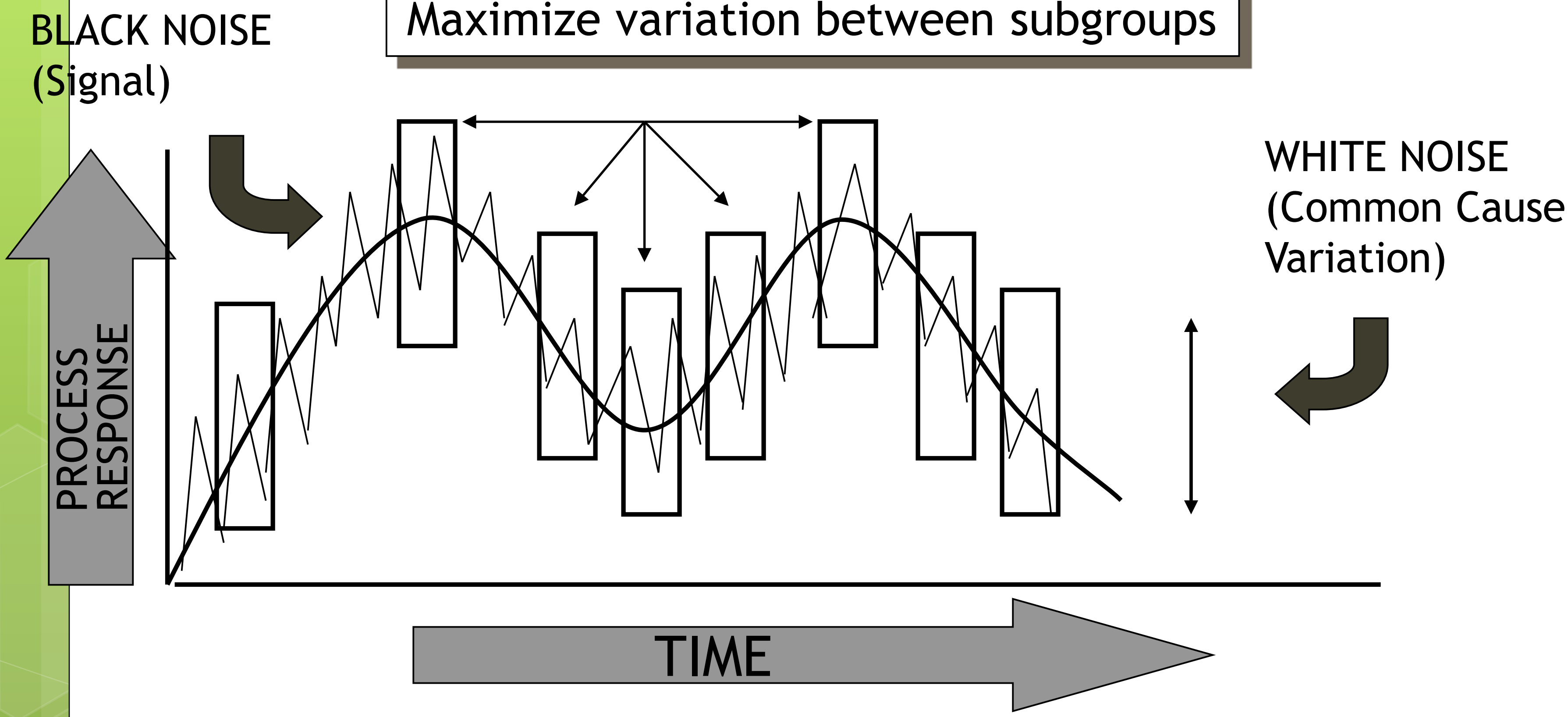


Fishbone



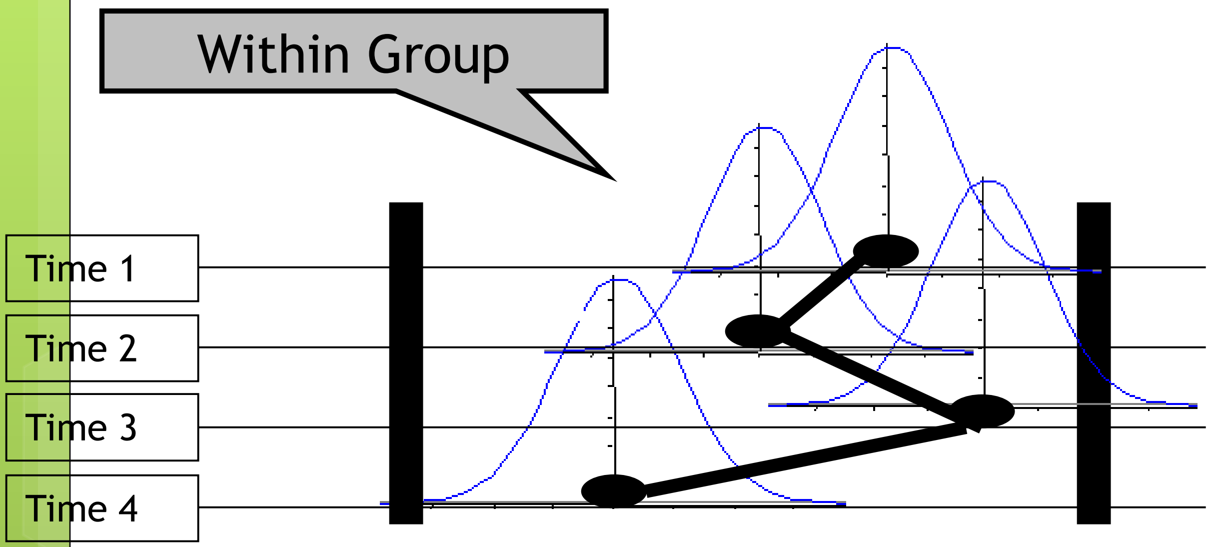
Historical Data

RATIONAL SUBGROUPS
Minimize variation within subgroups
Maximize variation between subgroups



RATIONAL SUBROUPING Allows samples to be taken that include only white noise, within the samples. Black noise occurs between the samples.

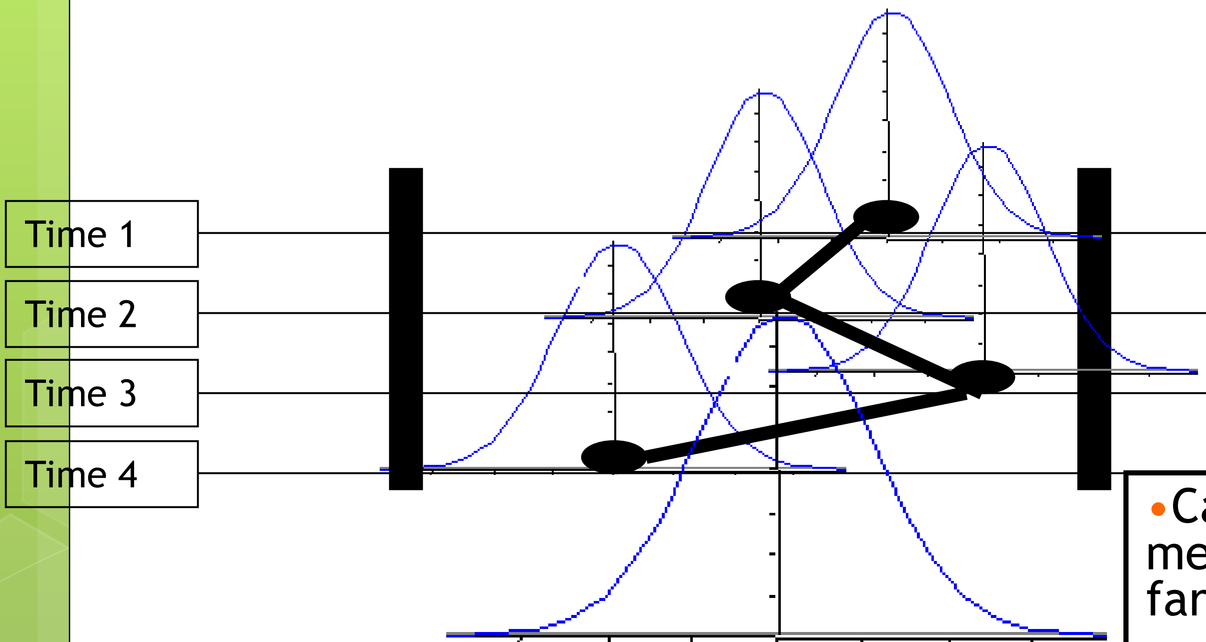
Visualizing the Causes



$\sigma_{st} + \sigma_{shift} = \sigma_{total}$

- Called σ short term (s_{st})
- Our potential – the best we can be
- The s reported by all 6 sigma companies
- The trivial many

Visualizing the Causes



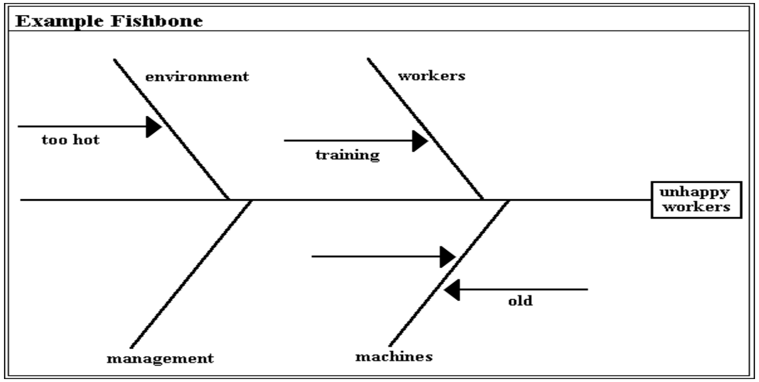
- Called σ_{shift} (truly a measurement in sigmas of how far the mean has shifted)
- Indicates our process control
- The vital few

$\sigma_{st} + \sigma_{shift} = \sigma_{total}$

Between Groups

- Assignable Cause

- Outside influences
- Black noise
- Potentially controllable
- How the process is actually performing over time



Fishbone

- Common Cause Variation

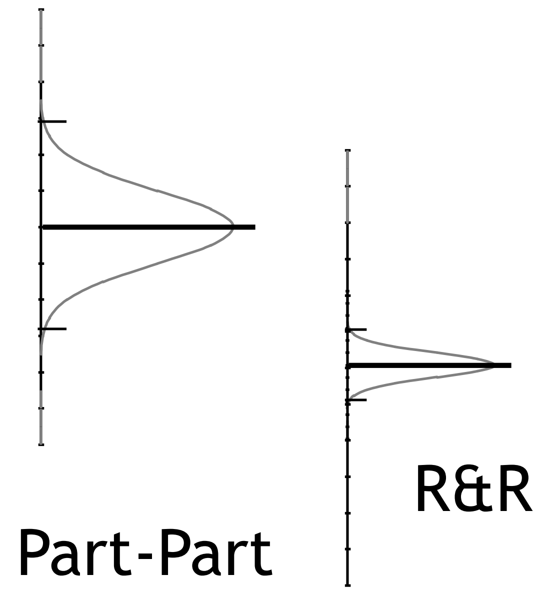
- Variation present in every process
- Not controllable
- The best the process can be within the present technology

Gauge R&R

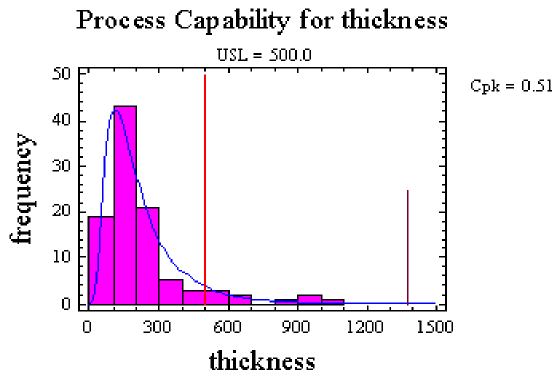
$$\sigma^2_{Total} = \sigma^2_{Part\text{-}Part} + \sigma^2_{R\&R}$$

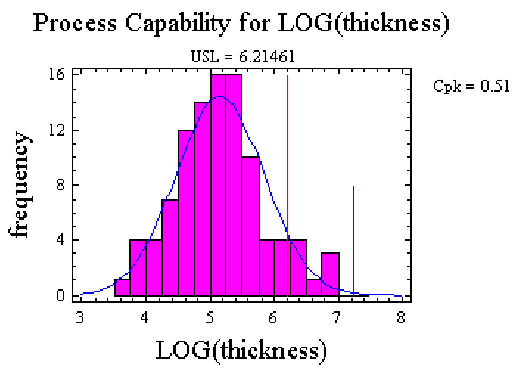
Recommendation:

Resolution £ 10% of tolerance to measure
Gauge R&R £ 20% of tolerance to measure

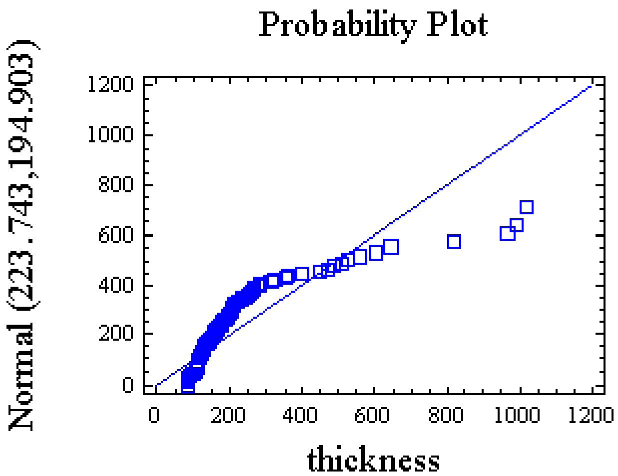


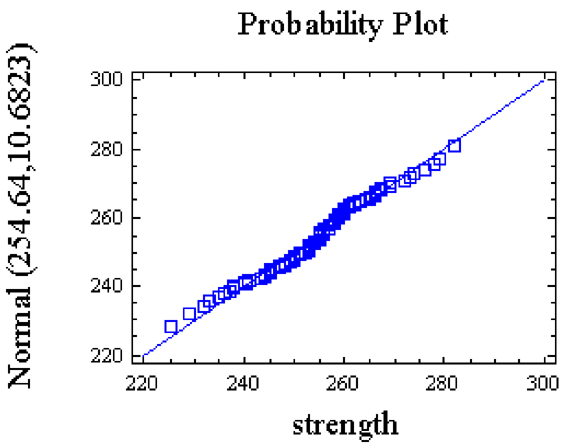
- ***Repeatability (Equipment variation)***
  - Variation observed with one measurement device when used several times by one operator while measuring the identical characteristic on the same part.
- ***Reproducibility (Appraised variation)***
  - Variation Obtained from different operators using the same device when measuring the identical characteristic on the same part.
- ***Stability or Drift***
  - Total variation in the measurement obtained with a measurement obtained on the same master or reference value when measuring the same characteristic, over an extending time period.



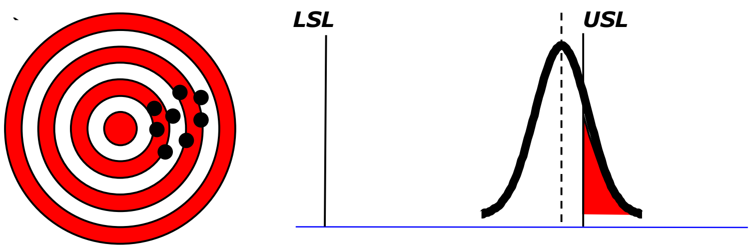


In many cases, the data sample can be transformed so that it is approximately normal. For example, square roots, logarithms, and reciprocals often take a positively skewed distribution and convert it to something close to a bell-shaped curve

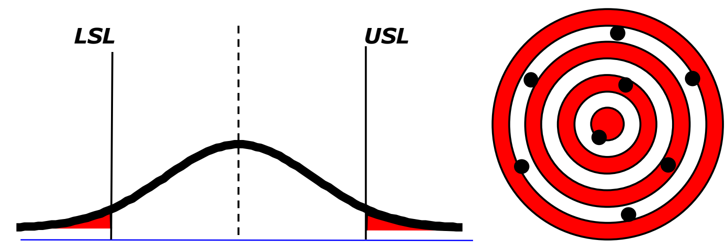




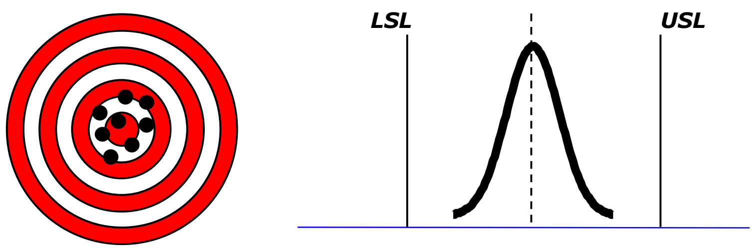
What do we Need?



Off-Target, Low Variation
High Potential Defects
Good Cp but Bad Cpk



On Target
High Variation
High Potential Defects
No so good Cp and Cpk



On-Target, Low Variation
Low Potential Defects
Good Cp and Cpk

- Variation reduction and process centering create processes with less potential for defects.
- The concept of defect reduction applies to **ALL** processes (not just manufacturing)

Eliminate "Trivial Many"

- Qualitative Evaluation
- Technical Expertise
- Graphical Methods
- Screening Design of Experiments

Identify "Vital Few"

- Pareto Analysis
- Hypothesis Testing
- Regression
- Design of Experiments

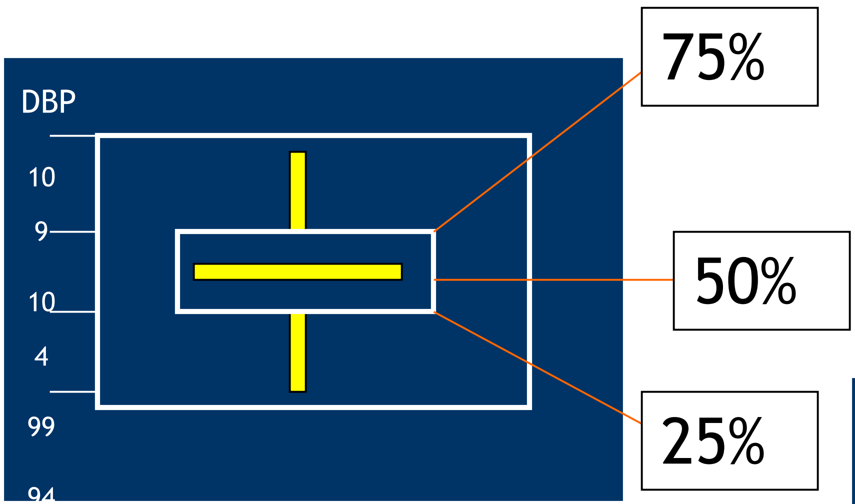
Quantify Opportunity

- % Reduction in Variation
- Cost/ Benefit

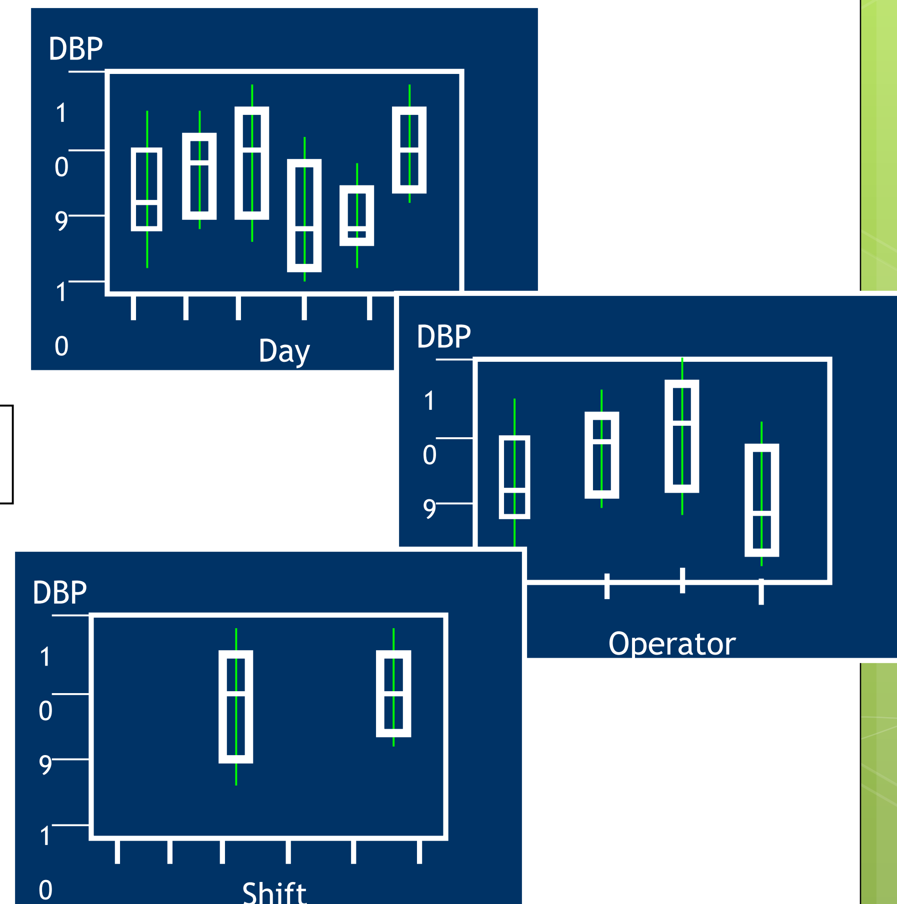
Our Goal:
Identify the Key Factors (x's)

Graph>Box plot

Without X values



Graph>Box plot

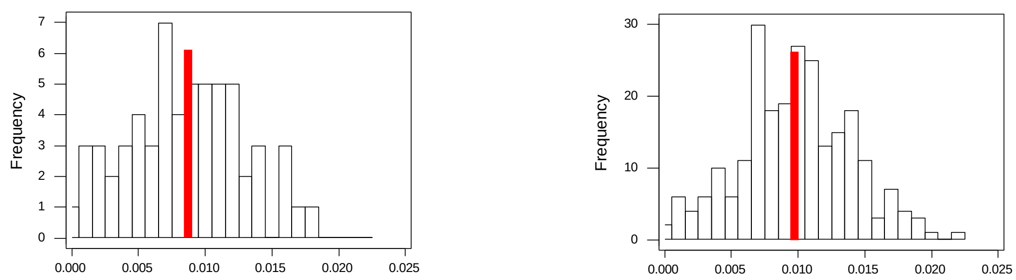


Box plots help to see the data distribution

Statistical Analysis

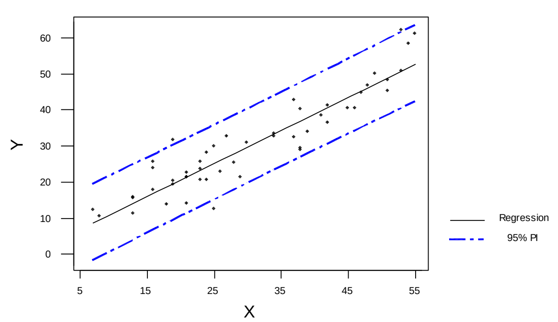
Apply statistics to validate actions & improvements

Hypothesis Testing



Compare Sample Means & Variances

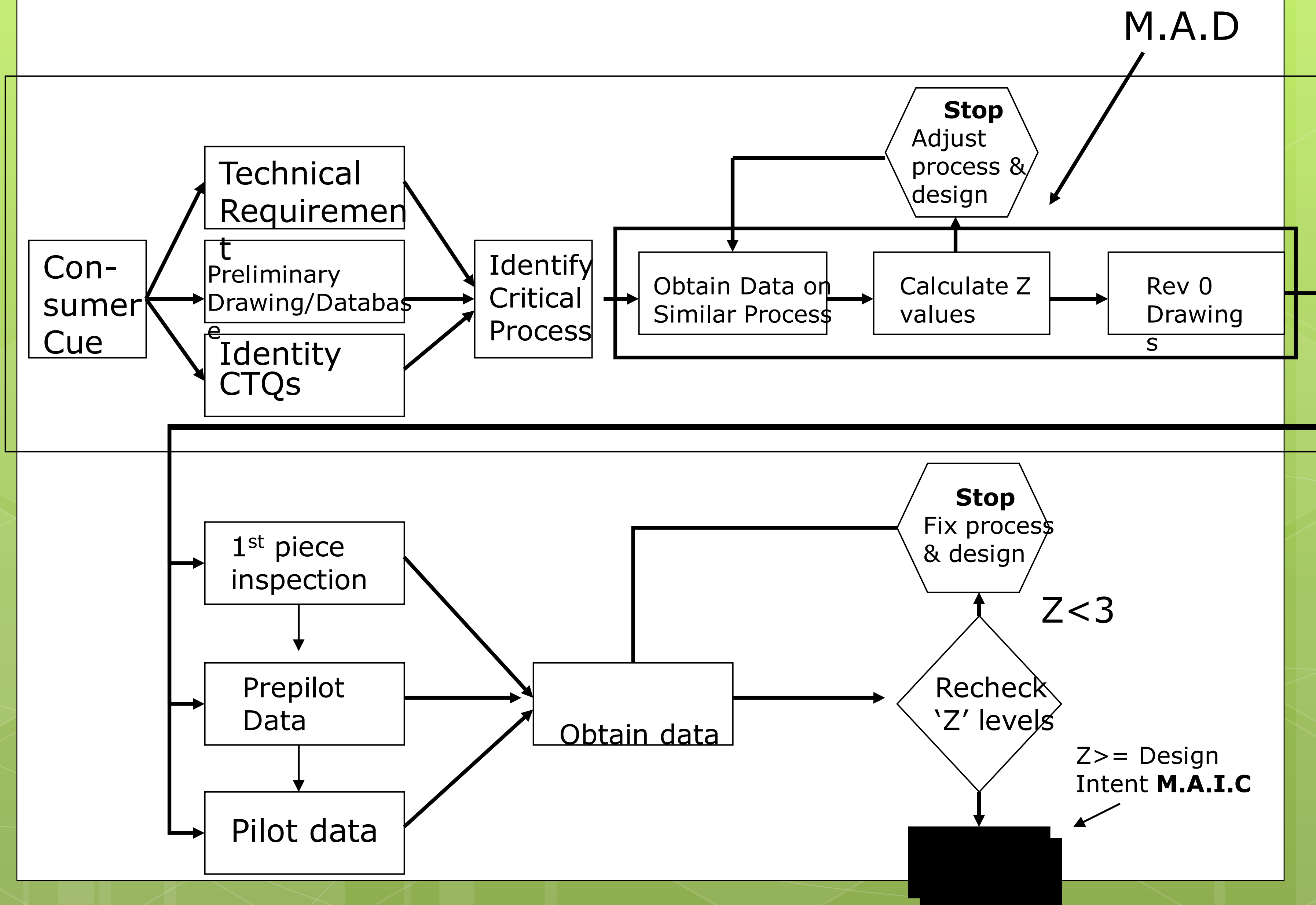
Regression Analysis



Identify Relationships

Establish Limits

- Is the factor really important?
- Do we understand the impact for the factor?
- Has our improvement made an impact
- What is the true impact?


M.A.D
Con-sumer Cue
Technical Requirement
Preliminary Drawing/Database
Identity CTQs
Identify Critical Process
Stop
Adjust process & design
Obtain Data on Similar Process
Calculate Z values
Rev 0 Drawings
1st piece inspection
Prepilot Data
Pilot data
Obtain data
Stop
Fix process & design
Z<3
Recheck 'Z' levels
Z>= Design Intent M.A.I.C
[redacted]

Excercise

$$x_{max} = \frac{v_0^2}{g} \cdot \sin(2 \cdot \alpha)$$

$$v_0 = \sqrt{\frac{x_{max} \cdot g}{\sin(2 \cdot \alpha)}}$$

$$g = \frac{v_0^2}{x_{max}} \cdot \sin(2 \cdot \alpha)$$

$$\alpha = \frac{1}{2} \cdot \sin^{-1}\left(\frac{x_{max} \cdot g}{v_0^2}\right)$$

Symbol	Beskrivelse	Enhed
x_{max}	Kastelængde	m (meter)
v_0	Begyndelseshastigheden	$\frac{m}{s}$
g	Tyngdeaccelerationen	$g \approx 9{,}82 \frac{m}{s^2}$
α	Affyringsvinklen	$^\circ$